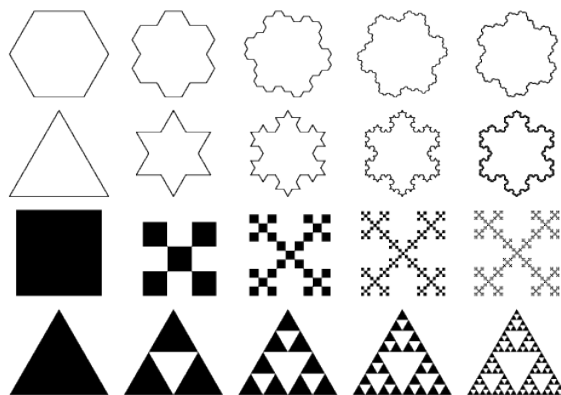


Bergen County Academies Math Team
 November 3rd, 2019 Mock (Practice) AMC 8

Welcome!

$$\begin{aligned}
 b &= a \\
 ab &= a^2 \\
 a^2 + ab &= a^2 + a^2 \\
 a^2 + ab &= 2a^2 \\
 a^2 + ab - 2ab &= 2a^2 - 2ab \\
 a^2 - ab &= 2a^2 - 2ab \\
 1(a^2 - ab) &= 2(a^2 - ab) \\
 1 &= 2
 \end{aligned}$$



- | | |
|--|----------------------|
| 2016 = $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 7$ | $40 \cdot 40 = 1600$ |
| 2017 = prime number | $41 \cdot 41 = 1681$ |
| 2018 = $2 \cdot 1009$ | $42 \cdot 42 = 1764$ |
| 2019 = $3 \cdot 673$ | $43 \cdot 43 = 1849$ |
| 2020 = $2 \cdot 2 \cdot 5 \cdot 101$ | $44 \cdot 44 = 1936$ |
| 2021 = $43 \cdot 47$ | 2019 |
| 2022 = $2 \cdot 3 \cdot 337$ | $45 \cdot 45 = 2025$ |
| 2023 = $7 \cdot 17 \cdot 17$ | $46 \cdot 46 = 2116$ |
| 2024 = $2 \cdot 2 \cdot 2 \cdot 11 \cdot 23$ | $47 \cdot 47 = 2209$ |
| 2025 = $3 \cdot 3 \cdot 3 \cdot 3 \cdot 5 \cdot 5$ | $48 \cdot 48 = 2304$ |
| 2026 = $2 \cdot 1013$ | $49 \cdot 49 = 2401$ |
| 2027 = prime number | $50 \cdot 50 = 2500$ |

2019 is a semi-prime number. A semi-prime number is a number that is the product of two prime numbers, not necessarily distinct. The next semi-prime number after 2019 is 2026.

1. In the following expression $2 \square 0 \square 1 \square 9$ each box is replaced by one of the arithmetic operators addition (+) or multiplication (\times). What is the sum of the possible different values for the expression it is possible to create?

- A.25 B.37 C.45 D. 48 E. 50

2. Mr. Plotnick recently switched his car insurance company to GEICO and saved 15%, his new semi-annual premium is \$2019 (rounded). How much was he paying with his previous insurance company?

- A. \$2,321.85 B. \$2753.92 C. \$2573.29 D. \$2375.29 E. \$2375.92

3. What is the first number larger than 2019 that has no proper composite factor?

- A. 2021 B. 2026 C. 2027 D. 2029 E. 2031

4. Two numbers are called *relatively prime* if they have no common factors other than 1. How many natural numbers less than or equal to 2019 are *relatively prime* to 2019.

- A. 1008 B. 1009 C. 1324 D. 1325 E. 2018

5. The natural numbers 1, 2, 3, 4, ..., 2019 are written concatenated in their natural order to form the single number 1234567891011....20182019. How many times does the number 1 appear?

- A. 1312 B. 1512 C. 1572 D. 1612 E. 1712

6. Suppose we multiply the consecutive even positive numbers $2 \times 3 \times 4 \times 6 \times 8 \times 10 \times \dots \times N$, where N is even. What is the smallest even number N such that this product is divisible by 2019?

- A. 674 B. 1010 C. 1346 D. 2020 E. 4038

7. Find the last digit in the full decimal representation of the number $\frac{1}{5^{2019}}$?

- A. 5 B. 2 C. 4 D. 6 E. 8

8. Find the value of the following sum

$$\left(\frac{1}{2019} + \frac{2}{2019} + \frac{3}{2019} + \dots + \frac{2018}{2019} \right)$$

- A. 1009 B. $1009 - \frac{1}{2019}$ C. $1009 + \frac{1}{2019}$ D. 1010 E. $1010 - \frac{1}{2019}$

9. A semi-prime number is a number that is the product of two prime numbers, not necessarily distinct. How many square-free semi-prime numbers are there less than 100?

- A. 30 B. 31 C. 32 D. 33 E. 35

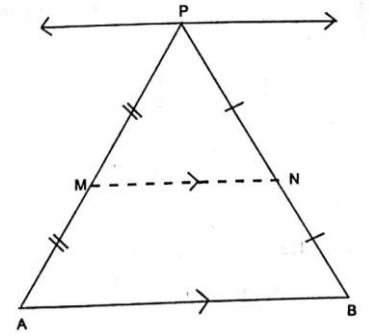
10. A lattice point is a point in the x-y coordinate plane such that both x and y coordinates are integers. A triangle $\triangle ABC$ in the coordinate plane has vertices B (673,0), C (0,3), and A at the origin (0,0). How many lattice points are completely inside the triangle $\triangle ABC$ (not including points on the boundary)?

- A.1342 B. 1344 C. 1346 D. 674 E. 1010

11. Suppose that $p \cdot q \cdot r \cdot s + t = 2019$, where $p, q, r, s,$ and t are distinct primes. Find the value of $p + q + r + s + t$.
- A. 100 B. 75 C. 73 D. 67 E. 50
12. The sum of all angles except for one of a convex polygon (all angles are less than 180°) is 2019° . What is the number of sides of the polygon?
- A. 13 B. 14 C. 15 D. 16 E. 17
13. If $4^{2019} + 7^{2019} - N$ is divisible by 9, where N is a single digit. What is the value of N ?
- A. 5 B. 1 C. 2 D. 4 E. 8
14. You visit an island inhabited by only Heroes and Villains. Heroes always tell the truth and Villains never tell the truth. You meet four inhabitants, A, B, C, and D. A says "Exactly one of us is a Villain"; and B says "We are all Villains." You ask C the question "Is A a Villain?" but the answer from C (yes or no), doesn't allow you determine if A is a Villain or not. How many Heroes are among A, B, C, and D?
- A. 1 or 3 B. 0 or 4 C. 2 or 3 D. 0 or 1 E. 3 or 4
15. If $S_n = 1 - 2 + 3 - 4 + 5 - 6 + \dots \pm n$, Find $|S_{2020} - S_{2019} + S_{2018}|$, where $|x|$ means the absolute value of x .
- A. 0 B. 1 C. 1010 D. 2019 E. 03029
16. Triangle $\triangle ABC$ is isosceles with base AC. Point P and Q are respectively within CB and AB such that $AC = AP = PQ = QB$. What is the measure of $\angle B$?
- A. 24° B. $24\frac{5}{7}^\circ$ C. 25° D. $25\frac{5}{7}^\circ$ E. $25\frac{4}{7}^\circ$
17. The ratio of the area of an equilateral triangle, a square, and a regular hexagon, all inscribed in the same circle is?
- A. $9:8\sqrt{3}:18$ B. $3:3\sqrt{2}:6$ C. $2:3\sqrt{3}:6$ D. $\sqrt{3}:2\sqrt{2}:2\sqrt{3}$ E. $\sqrt{3}:2:2\sqrt{3}$
18. A rectangle contains 3 circles, all tangent to the rectangle and also to each other. If the height of the rectangle is 4, find the value of the width of the rectangle?
- A. 5 B. $3 + 2\sqrt{2}$ C. $3 + 2\sqrt{3}$ D. $3 + \frac{3}{2}\sqrt{2}$ E. $5 - 2\sqrt{2}$
19. A ball was floating in a lake, when the lake froze. The ball was removed leaving a hole 24 inches across at the top and 8 inches deep? What is the radius of the ball?
- A. 10 B. $9\sqrt{2}$ C. $8\sqrt{2}$ D. 13 E. 21
20. Simon had to find the number of diagonals of a convex polygon and counted 2019 diagonals. Greg pointed out that this could not be correct. If Simon over-counted slightly, how many sides of the polygon were there?
- A. 63 B. 64 C. 65 D. 66 E. 67

21. In the figure, Points M and N are the midpoints of the sides PA and PB of triangle $\triangle PAB$, as point P varies along a line that is parallel to side AB, consider the following values:

- 1) The length of \overline{MN}
- 2) The area of triangle $\triangle PAB$
- 3) The area of trapezoid ABNM
- 4) The perimeter of triangle $\triangle PAB$



How many of these values are constant for any position of point P ?

- A. 0 B. 1 C. 2 D. 3 E. 4

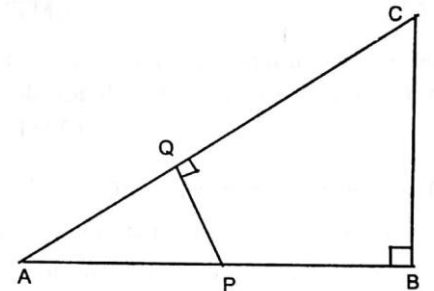
22. Suppose that a teacher gave a contest to 4 students, A, B, C, and D – and the teacher wants to let them grade each other's tests, but of course, no student should grade their own test. In how many ways can the teacher hand back the tests to the students for grading, such that no student receives their own test back?

- A. 6 B. 8 C. 9 D. 10 E. 12

23. In the figure, the angles $\angle Q$ and $\angle B$ are right angles.

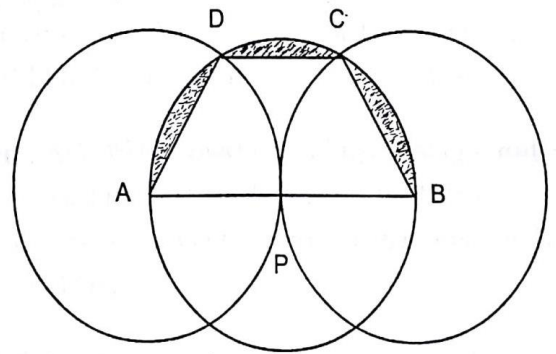
If $AQ = 15$, $BC = 16$, and $AP = 17$, find QC ?

- A. 19 B. 34 C. 13
D. 18 E. 31



24. AB is a line segment of length 4 inches. P is the midpoint of AB. Circles are drawn with centers A, P, and B and radii $AP = PB$. Find the area of the shaded region:

- A. $2\pi - \frac{3\sqrt{3}}{2}$ B. $2\pi - 2\sqrt{3}$ C. $2\pi - \frac{3\sqrt{2}}{2}$
D. $2\pi - 3\sqrt{2}$ E. $2\pi - 3\sqrt{3}$



25. In the figure, ABCD is a non-convex quadrilateral.

If $\angle BAD = 60^\circ$, $AB = 4$ and $BC = DC = 3$, find AC .

- A. $2\sqrt{3} + \sqrt{5}$ B. $2\sqrt{5} - \sqrt{3}$ C. $3\sqrt{3} - 2\sqrt{5}$
D. $2\sqrt{5} + \sqrt{3}$ E. $2\sqrt{3} - \sqrt{5}$

