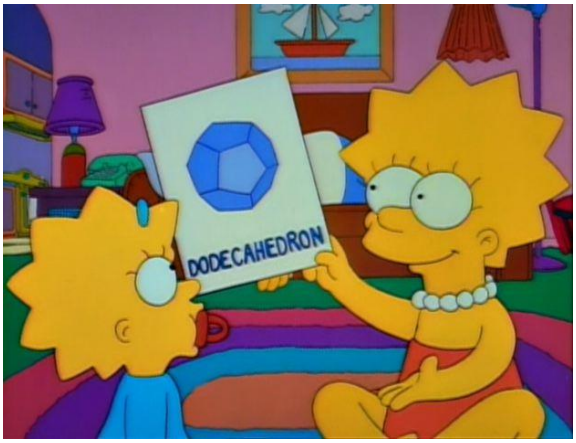
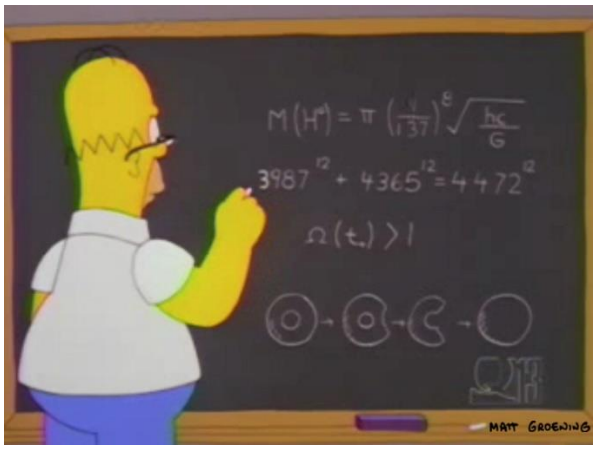
Bergen County Academies Math Team

November 4th, 2018 Mock (Practice) AMC 8

Welcome!





2017 = prime number
2018 = 2 · 1009
2019 = 3 · 673
2020 = 2 · 2 · 5 · 101
2021 = 43 · 47
2022 = 2 · 3 · 337
2023 = 7 · 17 · 17
2024 = 2 · 2 · 2 ·11 · 23
2025 = 3 · 3 · 3 · 3 · 5 · 5
2026 = 2 · 1013
2027 = prime number

2018 is a semi-prime number. A semi-prime number is a number that is the product of two prime numbers. The next semi-prime number after 2018 is 2019.

Daniel Plotnick

1. The smallest number N such that when added to 2018 gives a number that is divisible by 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 is

A. 52 B. 502 C. 702 D. 1002 E. 1762

2. A *sphenic* number is a positive integer with precisely 8 positive divisors. What is the smallest number that I can to add to 2018 to get a *sphenic* number?

A. 2 B. 3 C. 4 D. 5 E. 6

3. What is the last digit of $2018^1 + 2018^2 + \cdots + 2018^{2017} + 2018^{2018}$?

A. 0 B. 2 C. 4 D. 6 E. 8

4. An equilateral triangle with side length 3 is circumscribed by a circle, which in turn is circumscribed by a square. What is the area of the square?

A. 12 B. $12\sqrt{3}$ C. $12\sqrt{2}$ D. $9\sqrt{2}$ E. $6\sqrt{3}$

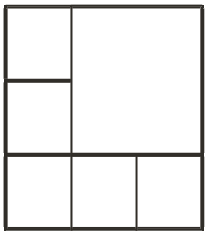
5. A square of side length 2 is circumscribed by a circle, which in turn is circumscribed by an equilateral triangle. What is the area of the equilateral triangle?

A. 12 B. $12\sqrt{3}$ C. $12\sqrt{2}$ D. $9\sqrt{2}$ E. $6\sqrt{3}$

6. Two numbers are called *relatively prime* if they have no common factors other than 1. How many natural numbers less than or equal to 2018 are *relatively prime* to 2018.

A. 1 B. 1008 C. 1009. D. 2017 E. none of these

7. You must color each square in the figure in Red, Green, or Blue. Any two squares with adjacent sides must be of a different color. In how many different ways can this coloring be done?



A. 9 B. 12 C. 18 D. 24 E. 30

8. Consider the set of all possible values of the expression $((a^b)^c)^d$ where a, b, c, and d take the values 2, 0, 1, 8 using each number only once. What is the most common value?

A. 64 B. 8 C. 2 D. 1 E. 0

9. Consider the set of all possible values of the expression $((a^b)^c)^d$ where a, b, c, and d take the values 2, 0, 1, 8 using each number only once. What is the difference between the largest possible value and the smallest possible value of the expression?

A. 63 B. 7 C. 6 D. 3 E. 1

10. Find the value of

$$\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots+\frac{1}{2018}\right)\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots+\frac{1}{2017}\right)-$$
$$\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots+\frac{1}{2018}\right)\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots+\frac{1}{2017}\right)$$

A. $\frac{2017}{2018}$ B. $\frac{2019}{2018}$ C. 1 D. $\frac{1}{2017}$ E. $\frac{1}{2018}$

11. Two standard 6 sided dice with numbers 1 through 6 on the sides are tossed. What is the probability that the square of the sum of the numbers showing is a 2 digit number?

A. $\frac{11}{12}$ B. $\frac{5}{6}$ C. $\frac{3}{4}$ D. $\frac{2}{3}$ E. $\frac{7}{12}$

12. Suppose a regular hexagon has a side D that is the same length as the diameter of a circle. What is the ratio of the area of the circle to the area of the hexagon?

A. $\frac{\pi\sqrt{3}}{18}$ B. $\frac{\pi\sqrt{3}}{9}$ C. $\frac{\pi}{9}$ D. $\frac{\pi}{4\sqrt{3}}$ E. $\frac{\pi\sqrt{3}}{6}$

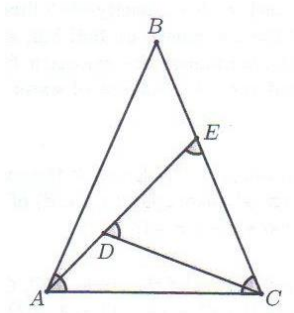
13. What is the sum of the prime factors of $1 + 2 + 3 + \cdots + 2018$?

A. 784 B. 1009 C. 1011 D. 1685 E. none of these

14. Suppose the equation $Ax + 2018 = 0$ is satisfied by some value of x, where -2018 < x < 2018. Which one of the following describes the possible values of A?

A. 0 < A < 1 B. A > 1 or A < -1 C. A > 1 D. A < -1 E. A > 1 or -1 < A < 0

15. In the diagram at right, if AD = 2, and AC = 6 and ∠BAC = ∠BCA = ∠DEC = ∠CDE, find the value of AB^2.



A. 36 B. 42 C. 45

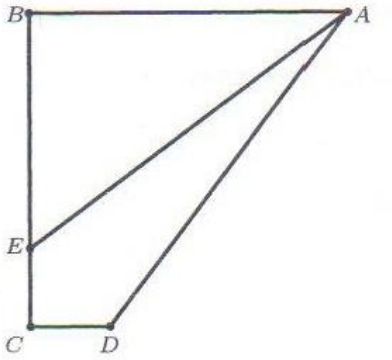
D. 48 E. 54

16. A *permutation* of four numbers (a, b, c, d) is an ordering using each number only once. For example, one permutation of (2, 0, 1, 8) is (1, 0, 8, 2). Supposed for a permutation (a, b, c, d) Jennifer calculates the list (a+b, b+c, c+d) to get 3 numbers (e,f,g) and then she calculates the list (e+f, f+g), to get 2 numbers (h, i), and then finally calculates the value h+i. What is the largest value she can obtain from any of the possible permutations of (2, 0, 1, 8) from this process?

A. 17 B. 27 C. 29

D. 31 E. 37

17. In the diagram, given ABCD is a right-angled trapezoid with AB = BC, ∠ABC = ∠BCD = 90° and E is a point on BC such that AE = AD. If AD = 10 and BE = 6, find the length of DE.



A. $\sqrt{2}$ B. $\sqrt{3}$ C. $\sqrt{6}$

D. $2\sqrt{2}$ E. $2\sqrt{3}$

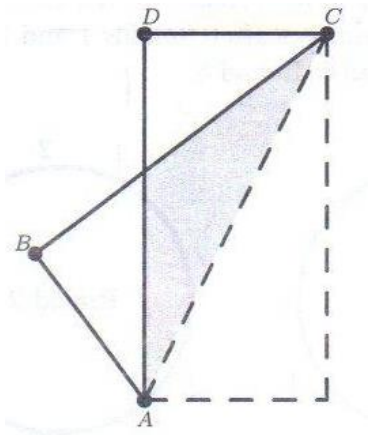
18. Let N = $2.\overline{018}$ where the bar over the decimal part means a repeating decimal, N= 2.018018018 … The value of N can be written in the form $A + \frac{B}{C}$. Where A, B, and C are natural numbers. What is the value of A + B + C?

A. 115 B. 116 C. 117 D. 118 E. 13

19. Find the value of $\sqrt{0^2+1} + \sqrt{1^2+3} + \sqrt{2^2+5} + \sqrt{3^2+7} + \cdots + \sqrt{61^2+123} + \sqrt{62^2+125}$?

A. 2016 B. 2017 C. 2018 D. 2080 E. 1953

20. Let ABCD be a rectangular sheet of paper with AB = 12 and BC = 24. If we fold the sheet of paper along the diagonal AC, there will be an overlapping region as shown in the diagram. What is the area of this overlapping region?



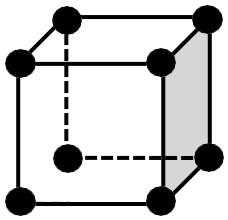
A. 72 B. $60\sqrt{2}$ C. 90

D. $60\sqrt{3}$ E. 234

21. Eleven consecutive positive integers are written on a board. Haneul erases one of the numbers. If the sum of the remaining numbers is 2018, what number did Haneul erase ?

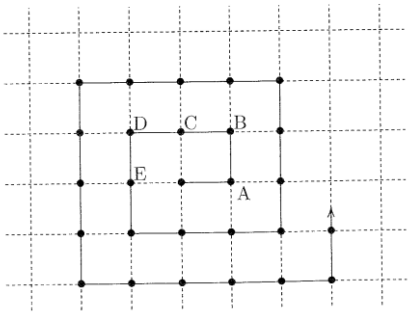
A. 202 B. 203 C. 204 D. 205 E. None of these

22. Shalin chooses eight of the nine numbers 2010, 2011, 2012, 2013, 2014, 2015, 2016, 2017, and 2018 and writes one at each vertex of the cube shown in the figure. Each number may be used only once, only one number can be used at each vertex and the sum of the four numbers on each side of the cube is 8056. Which one of the given numbers did he not use?



A. 2017 B. 2014 C. 2013 D. 2010 E. None of these

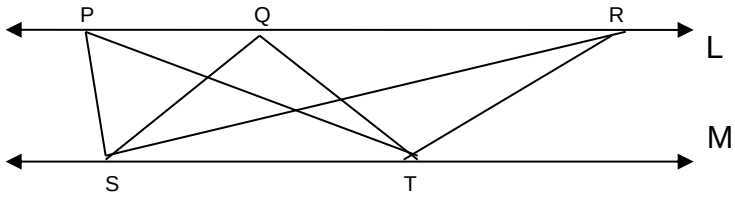
23. Julia walks a spiraling path on the Cartesian plane as follows: starting at the origin (0,0) and stopping at each point the first five stops are at A (1,0), B (1,1), C (0,1), D (-1,1), and E (-1,0), … her ninth stop is at the point (2,-1) and so on (see the diagram to the right.) What is the value of the sum x+y of the coordinates (x, y) at her 2018th stop?



A. -6 B. -22 C. 6 D. 22 E. none of the these

24. Ben and Wen play a game as follows. They each write down 3 positive integers that add up to 7 in non-decreasing order. The players then compare their choices in non-decreasing order and whoever has the higher number in the 1st position gets 1 point, 2nd position gets 2 points, and 3rd position 3 points. If a position is tied, no points are awarded. What numbers should either of them choose to maximize their probability of winning the game?

A. (1,1,5) B. (1,2,4) C. (1,3,3) D. (2,2,3) E. none of these



25. In the above figure, lines L and M are parallel. Points S and T are on line M and points P, Q and R are on line L. Three triangles are drawn, ΔPST, Δ QST, and Δ RST. Let the area of ΔPST be equal to a, the area of ΔQST be equal to b, and the area of Δ RST be equal to c. Which of the following inequalities is true?

A. a > b > c B. a > c > b C. c > a > b D. c > b > a E. None of these