1. While $10! = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2$ is an obvious place to start, that is too big.

We need divisibility by 9, and that covers divisibility by 3 also
We need divisibility by 8, and that covers divisibility by 2 and by 4.
We need divisibility by 7
We need divisibility by 5
Divisibility by 10 is covered by divisibility by 5 and by 2.
Divisibility by 6 is covered by divisibility by 3 and by 2.

So, we form the product $5 \cdot 7 \cdot 8 \cdot 9 = 40 \cdot 63$

$= 63 \cdot 40$

$$\begin{array}{r} 63 \\ 40 \\ \hline 2520 \end{array}$$

$2520 - 2018 = \boxed{502.}$ B

2. If a number is formed by the product of 3 prime numbers $p \cdot q \cdot r$, it will have the following 8 divisors:

$1, p, q, r, pq, pr, qr,$ and pqr

and no others.

So we are looking for the first number above 2018 with 3 prime factors. Consulting our cover sheet, we see that

$$2022 = 2 \cdot 3 \cdot 337$$

$$2022 - 2018 = \boxed{4}$$ Choice C.

3. We can ignore the 2000's part and just look for the last digits of

$$8^1 + 8^2 + 8^3 + \cdots + 8^{2017} + 8^{2018}$$

Why?

Look at the terminating digits of powers of 8

	LAST DIGIT	
$8^1 = 8$	8	NOTE THAT THIS PATTERN REPEATS
$8^2 = 64$	4	
$8^3 = 512$	2	
$8^4 = 4096$	6	
$8^5 = \ldots 8$	8	NOTE THAT $8+4+2+6 = 20$ ENDS IN ZERO.
$8^6 = \ldots 4$	4	
$8^7 = \ldots 2$	2	
$\vdots \quad 6$	6	

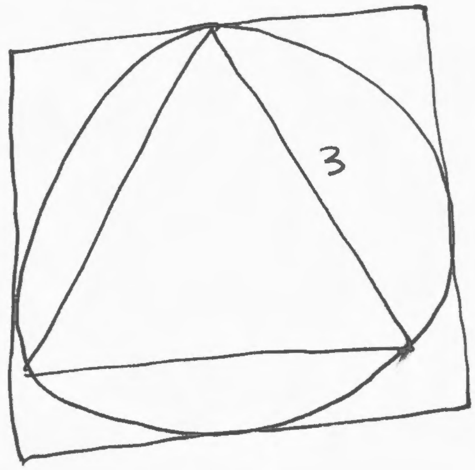
$$\text{So } \underbrace{8^1+8^2+8^3+8^4}_{\text{ends in } 0} + \cdots + \underbrace{8^{2013}+8^{2014}+8^{2015}+8^{2016}}_{\text{ends in } 0}$$

8^{2017} then ends in 8

8^{2018} then ends 4

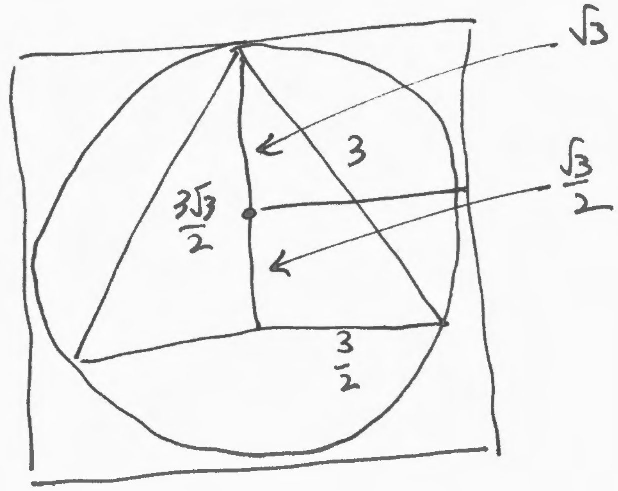
$8+4=12$ ends in $\boxed{2.}$ Choice B

4.



THIS IS THE DIAGRAM

USE THE PROPERTIES OF AN EQUILATERAL TRIANGLE.



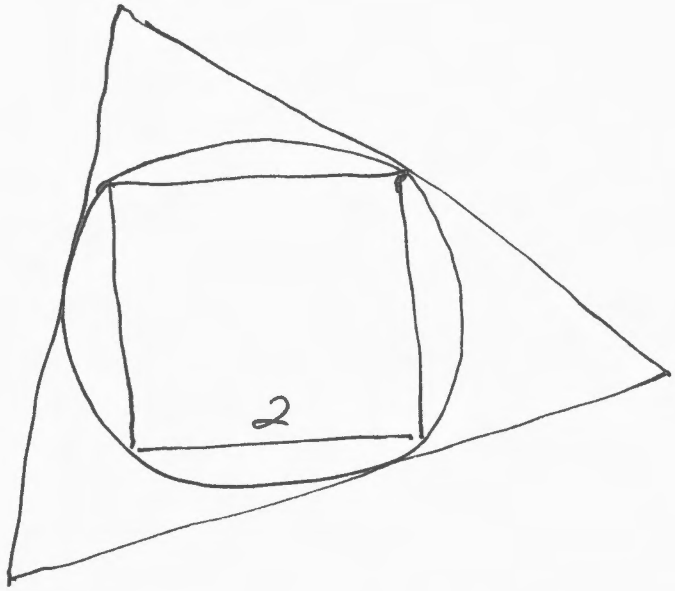
SO THE SIDE OF THE SQUARE IS $2\sqrt{3}$

THE AREA IS

$(2\sqrt{3})^2 = 4 \cdot 3 = \boxed{12}$

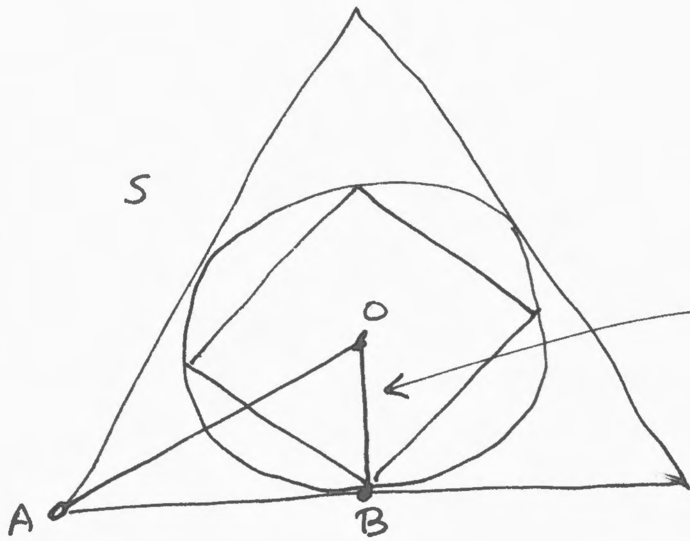
CHOICE A.

5.



THIS IS THE DIAGRAM.

USE THE PROPERTIES OF A SQUARE AND AN EQUILATERAL TRIANGLE.



$\sqrt{2}$. Angle OAB is $30°$, so $AO = 2\sqrt{2}$

Angle AOB is $60°$, so $AB = \sqrt{2} \cdot \sqrt{3} = \sqrt{6}$

AB is half the side of the equilateral triangle, so the side $s = 2\sqrt{6}$.

The area of an equilateral triangle is $\frac{s^2\sqrt{3}}{4}$

$$\frac{(2\sqrt{6})^2 \cdot \sqrt{3}}{4} = \frac{24 \cdot \sqrt{3}}{4} = \boxed{6\sqrt{3}}$$ CHOICE E.

6. $2018 = 2 \cdot 1009$, both primes.

Any number divisible by 2 is not relatively prime to 2018.

Any number divisible by 1009 is not relatively prime to 2018.

This leaves odd numbers not divisible by 1009

$1, 3, 5, 7, \ldots, 1007, 1011, \ldots, 2017$

So this is $1009 - 1 = \boxed{1008}$ numbers. Choice B.

For those more sophisticated, there is a formula for this

if $N = p^a \cdot q^b \cdot r^c \cdots$

where $p, q, r, \ldots$ are primes

then $\varphi(N) = N\left(1-\frac{1}{p}\right)\left(1-\frac{1}{q}\right)(\cdots)$

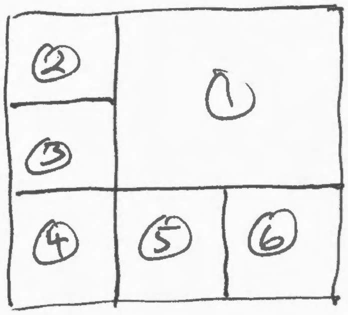
$$\varphi(2018) = 2018\left(1-\frac{1}{2}\right)\left(1-\frac{1}{1009}\right)$$

$$= 2018\left(\cdot\frac{1}{2}\right)\left(\frac{1008}{1009}\right)$$

$$= 2018 \cdot \frac{1008}{2018}$$

$$= 1008.$$

7.



There are 3 possibilities for the largest square labeled 1. Any choice of the remaining 2 colors for square 2 determines the entire coloring through 6.

$$3 \cdot 2 = 6$$

The other case is choosing the same color for 1 and 4, which are not adjacent. There are 3 possibilities again but this time there are two independent choices for 2 and 3 and for 5 and 6 with the remaining 2 colors.

$$3 \cdot 2 \cdot 2 = 12$$

$6 + 12 = \boxed{18}$. Choice C.

8. You could calculate all possibilities, there are
24 possible permutations of 2, 0, 1, 8.
But it is easier to note that if a is zero,
then for any of the 6 possible permutations of 1, 2, 8
the value is still 0.
For the remaining 18 possibilities one of b, c, d is 0,
which reduces the expression to 1 no matter what. Why?
So the most common value is $\boxed{\text{1. Choice D}}$

9. From the previous question, we know the expression only takes the value 1 or 0.

The difference is $\boxed{\text{1. Choice E.}}$

10. Find the value of

$$\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots+\frac{1}{2018}\right)\left(1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{2017}\right)$$
$$-\left(1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{2018}\right)\left(\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{2017}\right)$$

It is obviously too complicated to calculate, there must be another way. In fact two ways.

FIRST. Suppose $x=\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots+\frac{1}{2017}$,

then the expression is

$$\left(x+\frac{1}{2018}\right)(1+x)-\left(1+x+\frac{1}{2018}\right)(x)$$

OR

$$\left(x+x^2+\frac{1}{2018}+\frac{x}{2018}\right)-\left(x+x^2+\frac{x}{2018}\right)$$

EVERYTHING CANCELS except for the $\frac{1}{2018}$. Choice E.

SECOND. FIND A PATTERN,

Suppose it just went to N instead of 2018

$N=2$ $\quad \left(\frac{1}{2}+\frac{1}{3}\right)\left(1+\frac{1}{2}\right)-\left(1+\frac{1}{2}+\frac{1}{3}\right)\left(\frac{1}{2}\right)$

$$\frac{5}{6}\cdot\frac{3}{2}-\frac{11}{6}\cdot\frac{1}{2}$$

$$\frac{15}{12}-\frac{11}{12}=\frac{4}{12}=\frac{1}{3}$$

$N=3$ $\quad \left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)\left(1+\frac{1}{2}+\frac{1}{3}\right)-\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)\left(\frac{1}{2}+\frac{1}{3}\right)$

$$\frac{13}{12}\cdot\frac{11}{6}-\frac{25}{12}\cdot\frac{5}{6}$$

$$\frac{143}{72}-\frac{125}{72}=\frac{18}{72}=\frac{1}{4}$$

...

Hmm. PATTERN

$N=2018$. $\quad \frac{1}{2018}$.

16. DRAW THE SAMPLE SPACE

	1	2	3	4	5	6
1	4	9	16	25	36	49
2	9	16	25	36	49	64
3	16	25	36	49	64	81
4	25	36	49	64	81	100
5	36	49	64	81	100	121
6	49	64	81	100	121	144

1 DIGIT

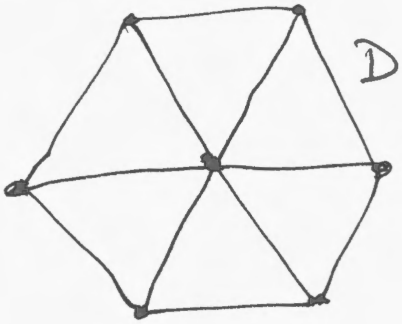
3 DIGITS.

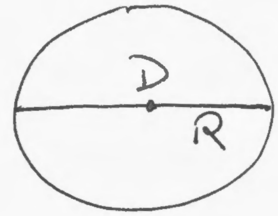
36 POSSIBILITIES

27 HAVE TWO DIGITS

$$\frac{27}{36} = \frac{3}{4}$$

12.





$AREA = \pi R^2$

$RADIUS = \frac{D}{2}$

A regular hexagon is made up of 6 equilateral triangles.

The area of an equilateral triangle with side D is $\frac{D^2\sqrt{3}}{4}$

The area of the hexagon is $\frac{6 \cdot D^2\sqrt{3}}{4}$

The area of the circle is $\left(\frac{D}{2}\right)^2 \cdot \pi = \frac{D^2\pi}{4}$

THE RATIO IS

$$\frac{\frac{D^2\pi}{4}}{\frac{6D^2\sqrt{3}}{4}} = \frac{D^2\pi}{6D^2\sqrt{3}} = \frac{\pi}{6\sqrt{3}}$$

$$= \frac{\pi\sqrt{3}}{6\sqrt{3}\cdot\sqrt{3}} = \boxed{\frac{\pi\sqrt{3}}{18}}$$

CHOICE A.

13. IMPORTANT FORMULA FOR THE SUM OF THE FIRST N NUMBERS

$$1+2+3+\cdots+N = \frac{N(N+1)}{2}$$

THIS CAN BE PROVED IN A VARIETY OF WAYS.

$$1+2+3+\cdots+2018 = \frac{2018\cdot 2019}{2}$$

$$= 1009\cdot 2019$$

Conveniently, our cover sheet has the prime factorization of 2019

$$2019 = 3\cdot 673$$

So, the sum is $1009\cdot 3\cdot 673$

$$1009+673+3 = \boxed{1685}$$ Choice D.

14.

$$Ax + 2018 = 0$$

$$Ax = -2018$$

$x = -\frac{2018}{A}$, and we want $-2018 < x < 2018$

$$-2018 < -\frac{2018}{A} < 2018$$

DIVIDE THROUGH BY 2018, no change in inequalities

$$-1 < \frac{-1}{A} < 1$$

$-1 < -\frac{1}{A}$ $\qquad$ $-\frac{1}{A} < 1$

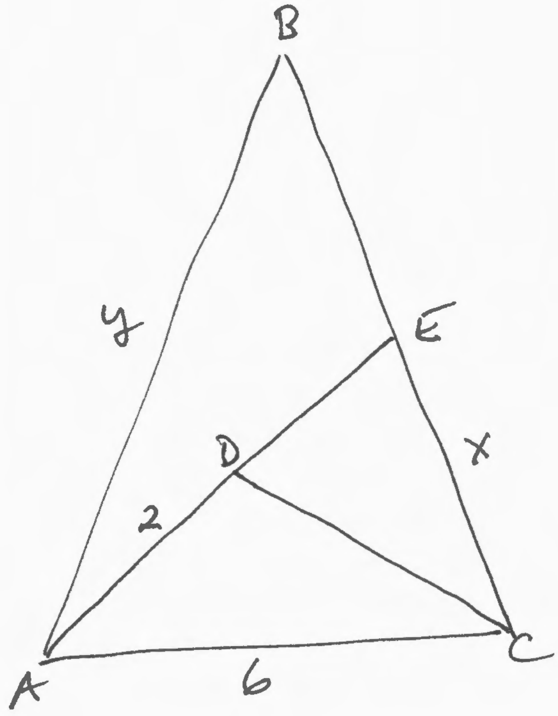
$1 > \frac{1}{A}$ $\qquad$ $\frac{1}{A} > -1$ multiply by -1, reverse inequality

$A > 1$ $\qquad$ $A < -1$ invert, reverse inequality

$$\boxed{A > 1 \text{ or } A < -1 \quad \text{Choice B.}}$$

The other way to do it is to try values of A for each of the choices, choosing $A = \frac{1}{2}$ and $A = -\frac{1}{2}$ allows you to narrow the choices, choosing $A = 2$ and $A = -2$ confirms that you can solve it.

15.



We have three similar triangles,

$\triangle ABC \sim \triangle CDE \sim \triangle EAC$

The triangles are isosceles,

so $DE = 4$.

Suppose $EC = x$, $AB = y$

Then from

$\triangle CDE \sim \triangle EAC$

$\frac{x}{6} = \frac{4}{x}$ so $x^2 = 24$

From $\triangle ABC \sim \triangle EAC$

$$\frac{y}{6} = \frac{6}{x}, \text{ so } y = \frac{36}{x}$$

$$y^2 = \frac{36^2}{x^2} = \frac{36^2}{24} = \frac{36}{24} \cdot 36$$

$$= \frac{3}{2} \cdot 36 \boxed{= 54}$$ Choice E.

16. One way to do it is to write out all 24 permutations of $(2,0,1,8)$ and check the values

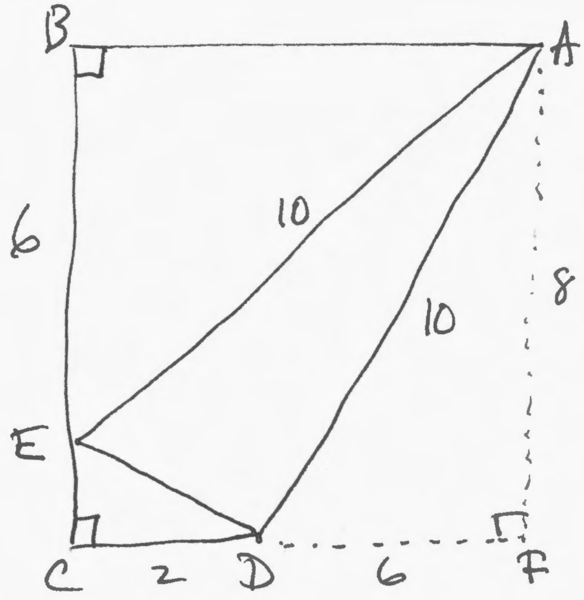
More cleverly, note that the calculations are as follows

$$(a, b, c, d)$$
$$\downarrow$$
$$(a+b,\ b+c,\ c+d)$$
$$\downarrow$$
$$(a+2b+c,\ b+2c+d)$$
$$\downarrow$$
$$a+3b+3c+d$$
$$a+3(b+c)+d$$

To maximize this, we want to maximize $b+c$, $b+c=10$.

$$a+30+d$$
$$0+30+1=\boxed{31.\ \text{Choice D.}}$$

17.



ΔABE is a right triangle.

$AE = 10$

so $AB^2 + 6^2 = 10^2$

$AB = 8.$

But $AB = BC$, so $EC = 2.$

then the figure is a square

ΔAFD is also right so $DF^2 + 8^2 = 10^2$ and $DF = 6$

so $CD = 2$. ECD is a right triangle and

$ED^2 = 2^2 + 2^2 = 4$

$ED = \boxed{2\sqrt{2} \text{ Choice D}}$.

14. Find the value of

$$\sqrt{0^2+1} + \sqrt{1^2+3} + \sqrt{2^2+5} + \sqrt{3^2+7} + \dots + \sqrt{61^2+123} + \sqrt{62^2+125}$$

$$\downarrow$$

$$\sqrt{1} + \sqrt{4} + \sqrt{9} + \sqrt{16} + \dots + \sqrt{61^2+123} + \sqrt{62^2+125}$$

LOOKS LIKE SQUARES

$$\sqrt{A^2+(2A+1)} = \sqrt{A^2+2A+1}$$
$$= \sqrt{(A+1)^2}$$
$$= A+1$$

So the sequence is

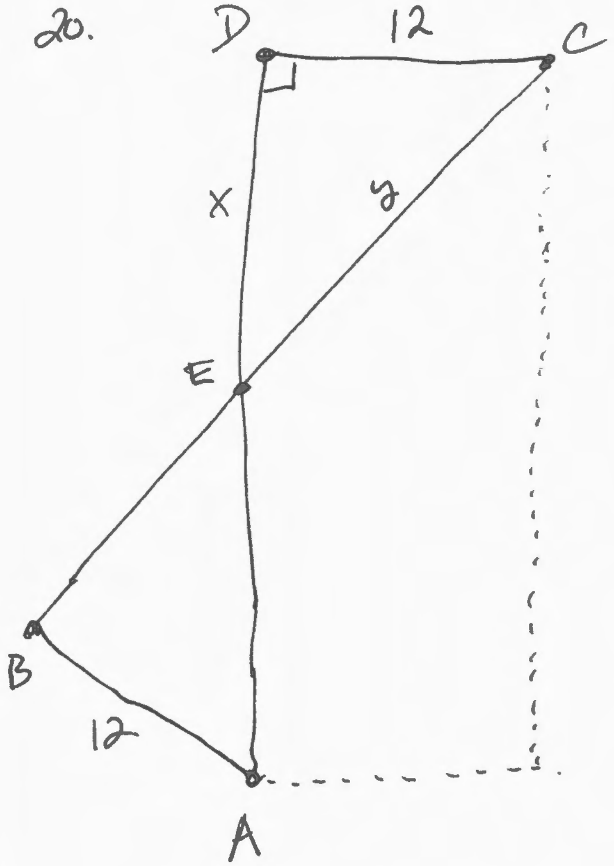
$$\sqrt{1} + \sqrt{4} + \sqrt{9} + \sqrt{16} + \dots + \sqrt{61^2} + \sqrt{62^2} + \sqrt{63^2}$$

$$= 1 + 2 + 3 + 4 + \dots + 61 + 62 + 63$$

$$= \frac{63\cdot 64}{2} = 63\cdot 32 = \boxed{2016.}$$ Chose A.

$$\begin{array}{r} 63 \\ \times\ 32 \\ \hline 126 \\ 189 \\ \hline 2016 \end{array}$$

20.



$AB = 12, \ BC = 24$

So $CD = 12$

Let $DE = x$, $CE = y$

Then, by the Pythagorean Theorem in ΔCDE

$$x^2 + 12^2 = y^2$$

or

$$12^2 = y^2 - x^2$$

if $CE = y$, then $BE = 24 - y$

if $DE = x$, then $AE = 24 - x$

By the Pythagorean Theorem in ΔABE

$$12^2 + (24-y)^2 = (24-x)^2$$

$$12^2 + 24^2 - 2\cdot 24\cdot y + y^2 = 24^2 - 2\cdot 24\cdot x + x^2$$

Moving things around and cancelling the 24^2

$$12^2 + 2\cdot 24\cdot x - 2\cdot 24 y = x^2 - y^2 = -12^2$$

$$2\cdot 12^2 = 2\cdot 24 y \cdot 2\cdot 24 x$$

$$12 = 2y - 2x$$

$$6 = y - x \quad \text{or} \quad y = 6 + x$$

In triangle CDE,

$$12^2 + x^2 = (6+x)^2$$

$$144 + x^2 = 36 + 12x + x^2$$

$$108 = 12x$$

$9 = x$, so the area of $\Delta CDE = \frac{9\cdot 12}{2} = 54$

The area of the overlapping region is $\frac{12\cdot 24}{2} - 54 = 144 - 54 = 90.$

21. Let the numbers be

$x-5,\ x-4,\ x-3,\ x-2,\ x-1,\ x,\ x+1,\ x+2,\ x+3,\ x+4,\ x+5$

These are consecutive numbers

If we add up these 11 numbers, we get $11x$

if we omit one, say $(x+k)$, where k is one of $-5, -4, \dots, 4, 5$.

then the problem says

$$11x - (x+k) = 2018$$

or $$10x - k = 2018$$

$$10x = 2018 + k$$

so k must be 2.

$$10x = 2020$$

$$x = 202,\quad k = 2$$

and the omitted number is $202 + 2 = 204$.

$\boxed{\text{Choose C.}}$

22. We can ignore the 2000s part and just deal with 10 through 18 to reduce the numbers.

If we add up the numbers on all faces we have to get $6 \cdot 56 = 336$

each number is used 3 times.

if we used all the numbers 3 times

$3(10+11+\cdots+17+18)$

$= 3 \cdot \left(9 \cdot 9 + \frac{9 \cdot 10}{2}\right)$

$= 3 \cdot (81+45) = 3(126) = 378$

which exceeds 336 by 42

$\frac{42}{3} = 14$, so 14 was not used

so in the original problem 2014 was not used $\boxed{\text{Choice B}}$

23. Find a pattern.

Let's look at the coordinates on the "diagonal"

At point $B(1,1)$, the distance travelled is $1+1$

At point $F(-1,-1)$, the distance travelled is $1+1+2+2$

At point $(2,2)$, the distance travelled is $1+1+2+2+3+3$

At point $(-2,-2)$, the distance travelled is $1+1+2+2+3+3+4+4$

... and so on.

Turning it around, after traveling

$1+1+2+2+3+3+\cdots+N+N$, for N even, we are at $\left(-\frac{N}{2}, \frac{-N}{2}\right)$

$$\text{Now } S = 1+1+2+2+3+3+\cdots+N+N$$

$$S = 2(1+2+3+\cdots+N)$$

$$S = 2\left(\frac{N(N+1)}{2}\right) = N(N+1)$$

We are looking for something near 2018 steps.

if $N=44$, $N(N+1) = 44 \cdot 45 = 1980$

so it is 1980 steps to point $(-22,-22)$.

The next steps are to the right along y coordinate -22, increasing x.

38 more steps puts us at $(16,-22)$ and we have

completed 2018 steps. $16+(-22) = -6$.

Choice A.

24.

Ben \ Wen	(1,1,5)	(1,2,4)	(1,3,3)	(2,2,3)
(1,1,5)	tie	tie	tie	WEN WINS
(1,2,4)	tie	tie	tie	tie
(1,3,3)	tie	tie	tie	tie
(2,2,3)	BEN WINS	tie	tie	tie

~~Any~~ Only (2,2,3) does better than tie. $\boxed{\text{Choice D.}}$

There are only 4 possible choices for the numbers.

25. TRICK QUESTION.

All the triangles have the same area.

The lines are parallel, so the triangle have the same height.

The base is the same ST for each triangle.

$\boxed{\text{Choice E.}}$ None of These.