1. Olivia will get 403 nickels with 2 pennies leftover.
She will get 201 dimes for the nickels with 1 nickel leftover.
She will get 80 quarters for the dimes with 1 dime leftover.
She will get 20 dollars for the quarters and she is left
with 1 dime, 1 nickel, and 2 pennies. or $\boxed{17¢ = 10¢ + 5¢ + 2¢.}$

2. $2^{2017}+0^{2017}+1^{2017}+7^{2017}$

Look at the last digits of powers of 2

$2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, \ldots$

$2, 4, 8, 6, 2, 4, 8, 6, 2, 4, 8, 6$

$1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10 \quad 11 \quad 12$

They repeat every 4. 2^{4N} will end in 6, so 2^{2016} will end in 6, and then 2^{2017} will end in 2.

Look at the last digits of powers of 7

$7^0=1, 7, 49, 343, 2401, 16807, \ldots$

$1, 7, 9, 3, 1, 7, 9, 3, 1 \ldots$

$0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8$

They repeat every 4. 7^{4N} will end in 1, so 7^{2016} will end in 1, and the 7^{2017} will end in 7

0^{2017} ends in 0

1^{2017} ends in 1

So the expression ends in $2+0+1+7=10$, so $\boxed{0.}$

3. $1+2+3+\cdots+2017$

The formula is $1+2+\cdots+N = \frac{N(N+1)}{2}$

$$1+2+3+\cdots+2017 = \frac{2017(2018)}{2}$$

$$= 2017 \cdot 1009$$

$$= 2{,}035{,}153$$

is closest to $\boxed{2{,}000{,}000}$

4. $82.5 = 78 + 4.5 = 6 \cdot 13 + 3 \cdot 1.5$

$99 = 96 + 3 = 6 \cdot 16 + 2 \cdot 1.5$

$114 = 6 \cdot 19$

$123 = 6 \cdot 20 + 3 = 6 \cdot 20 + 2 \cdot 1.5$

There is no way to make $\boxed{91}$

5. Which is largest

$$((1/2)/3)/4 = (1/6)/4 = 1/24$$

$$(1/2)/(3/4) = 1/2 \cdot 4/3 = 2/3$$

$$(1/(2/(3/4))) = (1/(8/3)) = 3/8$$

$$(1/(2/3))/4 = (3/2/4) = 3/8$$

$$\boxed{1/((2/3)/4) = 1/(2/12) = 1/(1/6) = 6/1 = \boxed{6}}$$

E is largest

6. The divisibility test for 4 is the last 2 digits

A2 must be divisible by 4, so

must be 12, 32, 52, 72, 92

A is 1, 3, 5, 7, or 9

The divisibility test for 3 is the sum of the digits

A+4 must be divisible by 3

$1+4=5$

$3+4=7$

$\boxed{5+4=9}$ A must be 5

$7+4=11$

$9+4=13$ The number is $\boxed{2052}$

7. There are 12 winners for the 6 games

$5+2+1 = 8$ ARE ACCOUNTED FOR

Indrani won 4 games.

8. Formally, let the consecutive numbers be $x-1, x, x+1$

$$(x+1)^2 - (x-1)^2$$
$$= (x^2+2x+1) - (x^2-2x+1)$$
$$= 4x$$

The only number divisible by 4 is $\boxed{2016}$

This can be detected by pattern also

1, 2, 3 $\quad 3^2-1^2 = 8 = 4 \cdot 2$

2, 3, 4 $\quad 4^2-2^2 = 12 = 4 \cdot 3$

3, 4, 5 $\quad 25-9 = 16 = 4 \cdot 4$

$\vdots$

9. 2017! How many zeros?

We need to count 5s (factors of 5)

5, 10, 15, 20, ..., 2015

We need to count 25s (double factors of 5)

25, 50, 75, ..., 2000

We need to count 125s (5^3)

125, 250, ..., 2000

We need to count 625s (5^4)

625, 1250, 1875

Another way to write this is

$$\left\lfloor \frac{2017}{5} \right\rfloor + \left\lfloor \frac{2017}{25} \right\rfloor + \left\lfloor \frac{2017}{125} \right\rfloor + \left\lfloor \frac{2017}{625} \right\rfloor$$

$$403 + 80 + 16 + 3$$

$$= 502$$

10. The sum of the factors of the sum of the factors of 2017 is

2017 is prime, so only has factors 1 and 2017

$1 + 2017 = 2018 = 2 \cdot 1009$, 1009 is prime also

the factors of 2018 then are

$1, 2, 1009, 2018$

$$\begin{array}{r} 2018 \\ 1009 \\ 2 \\ 1 \\ \hline \boxed{3030} \end{array}$$

11. Make a cross table of all possibilites
Remember to fill in the product

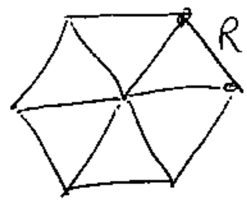
	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

prime or square circled

There are 36 possibilities
the successful outcomes are ~~shown~~ circled

$$\frac{14}{36} = \frac{7}{18}$$

12. A regular hexagon is made up of 6 equilateral triangles



The formula for the area of an equilateral triangle is Area A:

$$A = \frac{R^2\sqrt{3}}{4}$$

So the area of the hexagon is $\frac{6R^2\sqrt{3}}{4} = \frac{3R^2\sqrt{3}}{2}$

A circle of radius R has area πR^2

The ratio is $\dfrac{\pi R^2}{\frac{3R^2\sqrt{3}}{2}} = \dfrac{2\pi R^2}{3R^2\sqrt{3}}$

$$= \frac{2\pi}{3\sqrt{3}} = \frac{2\pi\sqrt{3}}{3\sqrt{3}\cdot\sqrt{3}}$$

$$= \boxed{\frac{2\pi\sqrt{3}}{9}}$$

13. There are 9 choices for the left digit, and thus only 3 places where the second copy can go. There are 9 possibilities the remaining digit.

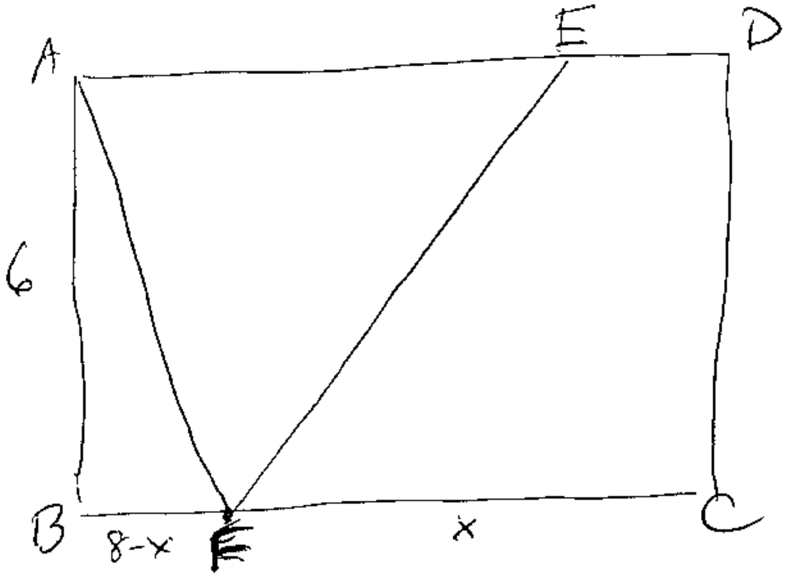
$$9 \times 3 \times 9 = 243$$

14. It says it has the same value for every a

Pick one. How about $a=2$.

$$\frac{3 \Omega 5}{1 \Omega 3} = \frac{3+4+5}{1+2+3} = \frac{12}{6} = \boxed{2}$$

15.



$AF = FC$

By the pythagorean theorem

$$6^2 + (8-x)^2 = AF^2$$

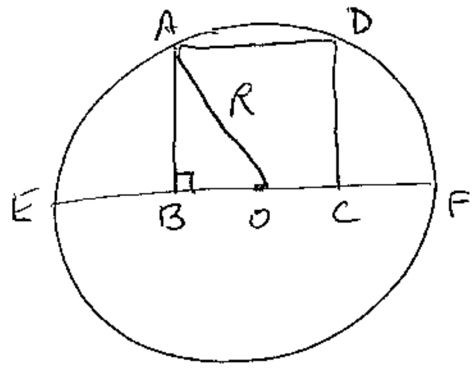
$$36 + 64 - 16x + x^2 = AF^2 = FC^2 = x^2$$

$$100 - 16x = 0$$

$$100 = 16x$$

$$\boxed{\frac{25}{4}} = \frac{100}{16} = x$$

16. Let the center of the circle be O



$AO = R =$ RADIUS OF CIRCLE

$AB^2 + BO^2 = R^2$

$AB = 17\sqrt{5}$

$BO = \frac{1}{2} AB = \frac{17\sqrt{5}}{2}$ since ABCD is a square.

$$(17\sqrt{5})^2 + \left(\frac{17\sqrt{5}}{2}\right)^2 = R^2$$

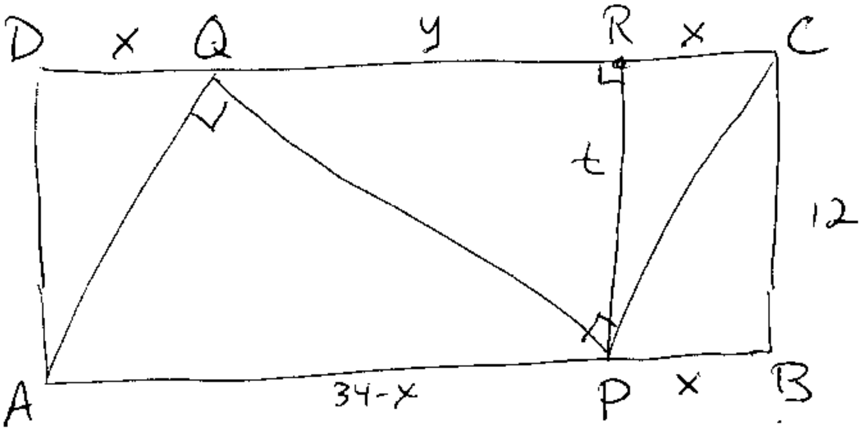
$$17^2 \cdot 5 + \frac{17^2 \cdot 5}{4} = R^2$$

$$\frac{17^2 \cdot 5^2}{4} = R^2$$

$$\frac{85}{2} = R$$

EF is a diameter $EF = 2R = 2 \cdot \frac{85}{2} = \boxed{85}$

17.



$\Delta ADQ \cong \Delta CBP$

34

In right triangle CPQ $t^2 = xy$ $(t=12)$

from similar triangles

$xy = 144$

x	y
1	144
2	72
4	36
6	24
$\boxed{8}$	$\boxed{18}$
$\boxed{9}$	$\boxed{16}$
12	12
16	9
18	8
24	6

But $2x+y=34$

$2\cdot 8+18=34$

$2\cdot 9+16=34$

x can be 8 or 9

So AP can be 26 or 25 $(=34-x)$

Can also create a quadratic, but a little out of scope.

18. Just do it!

1. $2017 \longrightarrow 4+1+49=54$
2. $54 \longrightarrow 25+16=41$
3. $41 \rightarrow 16+1=17$
4. $17 \rightarrow 49+1=50$
5. $50 \rightarrow 25+0=25$
6. $25 \rightarrow 5^2+2^2=29$
7. $29 \rightarrow 81+4=85$
8. $85 \rightarrow 64+25=89$
9. $89 \rightarrow 64+81=145$
10. $145 \rightarrow 1+16+25=42$
11. $42 \rightarrow 16+4=20$
12. $20 \rightarrow 4+0=4$
13. $4 \rightarrow 16=16$
14. $16 \rightarrow 36+1=37$
15. $37 \rightarrow 9+49=58$
16. $58 \rightarrow 64+25=89$
17. $89 \rightarrow 64+81=145$

⋮

It repeats every 8
Starting with the 9th
term - 89.

So
9, 17, 25, ... are all 89.

all $8N+1$ are 89

so 2017th is $\boxed{89}$

~~so 2017th [illegible]~~

$2017 = 8 \cdot 25$

$$\begin{array}{r} 252 \text{ R}1 \\ 8\overline{)2017} \\ 252 \\ 8 \\ \hline 2016 \end{array}$$

19. $N = 20.\overline{17}$

$100N = 2017.\overline{17}$

$99N = 1997$

$$N = \frac{1997}{99} = \frac{1980 + 17}{99} = 20 + \frac{17}{99}$$

$$= A + \frac{B}{C}$$

$$A + B + C = 20 + 17 + 99 = \boxed{136}$$

20.

A point is inside an odd number of circles if it is in the outermost ring (5), the thrid ring (3), or the middle circle (1).

The area of the middle circle is $\pi \cdot 1^2 = \pi$.

The thrid ring area is $\pi \cdot 3^2 - \pi \cdot 2^2 = 5\pi$

The 5^{th} outermost ring area is $\pi \cdot 5^2 - \pi \cdot 4^2 = 9\pi$

$$\pi + 5\pi + 9\pi = 15\pi$$

$$\boxed{m = 15}$$

21. The internal angles of the hexagon are 120° Let $AB = S =$ side

AD bisects $\angle CDE$ so $\angle ADC = 60^\circ$

$AC \perp CD$ so $\angle ACD = 90^\circ$

$\triangle ACD = 30\text{-}60\text{-}90$ triangle

Area of the hexagon is $\frac{6 \cdot s^2\sqrt{3}}{4}$

Area of $ACD = \frac{1}{2} CD \cdot AC = \frac{1}{2} s \cdot s\sqrt{3} = \frac{s^2\sqrt{3}}{2}$

$$\frac{6 \cdot s^2\sqrt{3}}{4} = N \cdot \frac{s^2\sqrt{3}}{2}$$

$$\frac{6}{4} = \frac{N}{2}$$

$$\boxed{3 = N}$$

22. What is

$$\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\cdots\left(1+\frac{1}{2016}\right)\left(1+\frac{1}{2017}\right)?$$

$$\frac{3}{2}\quad\frac{4}{3}\quad\frac{5}{4}\quad\cdots\quad\frac{2017}{2016}\quad\frac{2018}{2017}$$

TELESCOPING PRODUCT

$$=\frac{}{2}\quad\cdots\quad\frac{2018}{}$$

$$=\frac{2018}{2}=\boxed{1009}$$

23. Let D be the distance from Arthur's house to David's house. Then the actual ~~time~~ time will be the average of the two times.

So if S is the speed in miles per hour

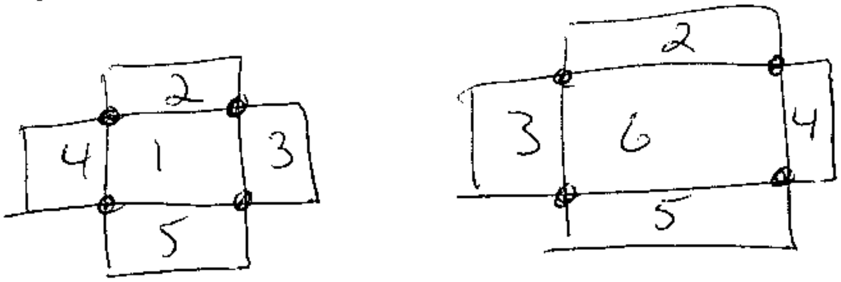
$$\frac{D}{S} = \frac{\frac{D}{60} + \frac{D}{90}}{2}$$

$$\frac{2}{S} = \frac{1}{60} + \frac{1}{90} = \frac{5}{180}$$

$$5S = 360$$

$$S = \boxed{72 \text{ miles/hour}}$$

24. Flatten out the cube



The dots are the corners. 8 corners

(1, 2, 3), (1, 2, 4), (1, 3, 5), (1, 4, 5)

(6, 2, 3), (6, 2, 4), (6, 3, 5), (6, 4, 5)

PRODUCTS ARE

6	8	15	20
36	48	90	120

The add up to $\boxed{\underline{343} = 7^3}$

25. If we let letters for the grid

A	B	C
D	E	F
G	H	I

$$S = (a+b+c)+(d+e+F)+(G+H+I) \quad \text{(rows)}$$
$$+(A+D+G)+(B+E+H)+(C+F+I) \quad \text{(cols)}$$
$$+(A+E+I)+(C+E+G) \quad \text{(diagonals}$$

$$= 4E+3A+3C+3G+3I+2B+2D+2F+2H$$

$$= 4E+3(A+C+G+I)+2(B+D+F+H)$$

to maximize set $E=9$

$$A, C, G, I = 8, 7, 6, 5$$
$$B, D, F, H = 4, 3, 2, 1$$

$$4 \cdot 9 + 3(8+7+6+5) + 2(4+3+2+1)$$
$$36 + 3 \cdot 26 + 2 \cdot 10$$
$$36 + 78 + 20$$
$$114 + 20 = \boxed{134}$$