

Bergen County Academies Sunday Math Team

Mock AMC 8

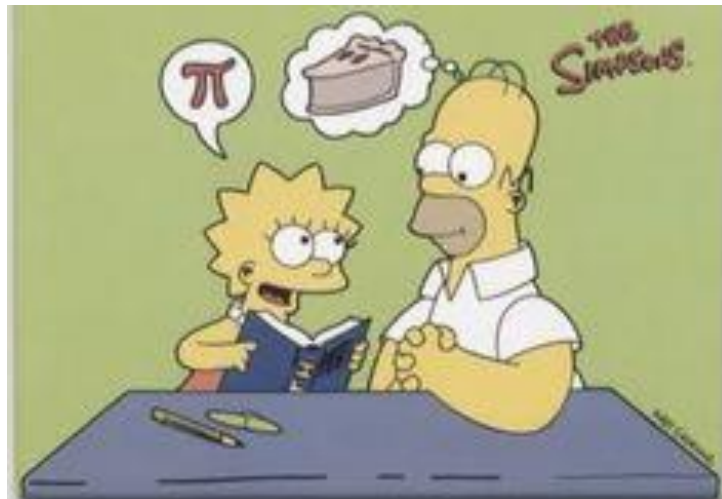
Sunday, November 15th, 2015

$$2013 = 3 \times 11 \times 61$$

$$2014 = 2 \times 19 \times 53$$

$$2015 = 5 \times 13 \times 31$$

$$2016 = 2^5 \times 3^2 \times 7$$



1. What is the value of $2015^2 - 2014 \times 2016$?

- A) -2015 B) -1 C) 0 D) 1 E) 2015

2. Which of the following numbers is divisible by 9?

- A) $10^{2015} + 3$ B) $10^{2015} + 5$ C) $10^{2015} + 6$ D) $10^{2015} + 7$ E) $10^{2015} + 8$

3. A cube has 6 faces, 8 vertices, and 12 edges. Suppose we assign one of the positive integers from 1 to 12 inclusive without duplication to each edge of a cube such that for any of the 6 faces, the numbers on the four edges of that face add up to the same value. What is that value?

- A) 15 B) 16 C) 20 D) 24 E) 26

4. What is the smallest value of N, such that if I add up the reciprocals of all the natural numbers from 1 to N inclusive, the value of the sum will be greater than 3?

- A) 12 B) 11 C) 10 D) 9 E) 8

5. The prime factorization of 2015 is $5 \times 13 \times 31$. 2015 is called a “square-free” number because its prime factorization only has prime factors with exponents 1. What is the least non-prime square-free number greater than 100.

- A) 101 B) 102 C) 103 D) 104 E) 105

6. What is the greatest fractional value less than 1 that I can add to the expression $\frac{2015}{2+0+1+5}$ to create an integral value?

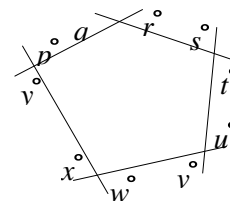
- A) $\frac{1}{2}$ B) $\frac{1}{5}$ C) $\frac{1}{6}$ D) $\frac{1}{7}$ E) $\frac{1}{8}$

7. What is the units digit of the sum $2^{2015} + 3^{2015} + 5^{2015}$?

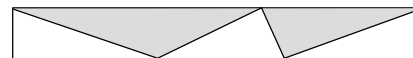
- A) 8 B) 6 C) 4 D) 2 E) 0

8. In the diagram at right, what is the sum $p + q + r + s + t + u + v + w + x + y$?

- A) 540 B) 720 C) 900 D) 1080 E) 1440



9. The rectangle shown is 3 units high and 14 units wide. What is the area of the shaded region?



- A) 14 B) 17 C) 21 D) 24 E) Cannot be determined.

10. Consider all the ways of ordering the 5 letters U, S, A, M, and O. Suppose these are listed in dictionary (alphabetical order), starting with AMOSU first. What position in the list would USAMO be?

- A) 97 B) 103 C) 109 D) 115 E) 120

11. On each of 9 consecutive days, Mr. Plotnick drank the following number of extra large coffees (extra cream, no sugar): 2, 0, 1, 5, 0, 2, 0, 5, 0. Which of the following is true of this set of numbers?

- A) median < mean < mode B) mean < median < mode
 C) median < mode < mean D) mode < median < mean
 E) mean < mode < median

12. What is the probability that if I pick a natural number between 1 and 500 (inclusive), it will divide into 2015 without remainder?

- A) $\frac{7}{500}$ B) $\frac{3}{250}$ C) $\frac{1}{100}$ D) $\frac{2}{125}$ E) $\frac{9}{500}$

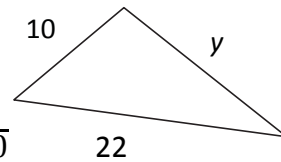
13. A square of side 1 inch is rotated 180° about one vertex in the plane. What is the area of the region swept out by this rotation in square inches?

- A) $\pi + 2$ B) $2\pi - 1$ C) $\pi + 1$ D) $2\pi - 2$ E) 4

14. Today, November 15th, 2015 is written numerically as 11/15/15. Notice that today's date only has 1s and 5s and 1s and 5s are digits of 2015. How many days of the year when written numerically only use digits from the year number 2015?

- A) 54 B) 55 C) 60 D) 66 E) 77

15. The triangle shown has an area of 88. What is the value of y ?



- A) 17.6 B) $2\sqrt{46}$ C) $8\sqrt{5}$ D) $13\sqrt{2}$ E) $6\sqrt{10}$

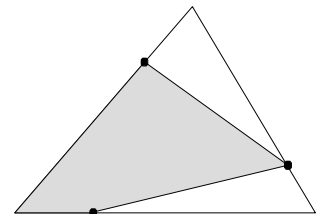
16. Find the value of $(6 + 7 + 8 - 9 - 10) + (11 + 12 + 13 - 14 - 15) + (16 + 17 + 18 - 19 - 20) + \dots + (2006 + 2007 + 2008 - 2009 - 2010) + (2011 + 2012 + 2013 - 2014 - 2015)$

- A) 403,809 B) 201,904 C) 401,802 D) 403,807 E) 807,618

17. Wen's agreement with his dad is that he spend less than half of his study time goofing on the internet. He currently spends $\frac{6}{7}$ of his study time goofing on the internet. If Wen currently studies for $3\frac{1}{2}$ hours every evening, how many minutes less each evening will Wen have to goof to meet his deal with his father?

- A) 84 B) 76 C) 75 D) 74 E) 42

18. A point is marked one quarter of the way along each side of a triangle, as shown. What fraction of the area of the triangle is shaded?



- A) $\frac{7}{16}$ B) $\frac{1}{2}$ C) $\frac{9}{16}$ D) $\frac{5}{8}$ E) $\frac{11}{16}$

19. 10 points are placed at random in a square of side length 1. Of the following values, for which is it always possible to find two points whose distance is at most the value?

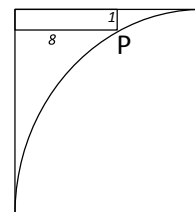
- A) $\frac{\sqrt{2}}{3}$ B) $\frac{\sqrt{2}}{2}$ C) $\frac{\sqrt{3}}{3}$ D) $\frac{1}{2}$ E) $\frac{\sqrt{10}}{10}$

20. Esther has taken 5 SAT subject tests and scored 800, 750, 780, 800, and 760. SAT subject test scores are always a multiple of 10. The minimum score is 200 and the maximum score of 800. 800 is perfect. Esther plans to take 4 more SAT subject tests before applying to college. Esther wants to improve her average SAT subject test score by 10 points. What is the minimum number of perfect scores she must get on her planned 4 additional subject tests to achieve her goal?

- A) 1 B) 2 C) 3 D) 4 E) Esther's plan can't succeed

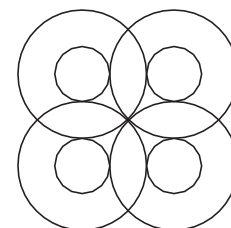
21. In the square as shown, a quarter-circle arc is drawn from one vertex to the opposite vertex. A point P on this arc is 8 units from one side of the square and 1 unit from the adjacent side of the square. What is the length of the side of the square?

- A) 11 B) 12 C) 13 D) 14 E) 15



22. The diagram at right shows eight circles of two different sizes. The circles are arranged in concentric pairs so that the centers form a square. Each larger circle touches one other larger circle and two smaller circles. The larger circles have radius 1. What is the radius of each smaller circle?

- A) $\frac{1}{3}$ B) $\frac{2}{5}$ C) $\sqrt{2} - 1$ D) $\frac{1}{3}\sqrt{2}$ E) $\frac{1}{2}$



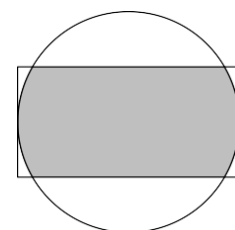
23. The number $\frac{2015}{2^{2015}}$ is represented as a decimal number. What is the third digit from the end?

- A) 8 B) 5 C) 0 D) 4 E) 2

24. Two equilateral triangles with side equal to 1 lie along the same baseline and overlap in a smaller equilateral triangle shaped region. If the overlapping region is $\frac{1}{11}$ of the total area of the region, what is the length of the altitude of this smaller equilateral triangle?

- A) $\frac{3}{11}$ B) $\frac{\sqrt{2}}{4}$ C) $\frac{\sqrt{11}}{11}$ D) $\frac{\sqrt{3}}{4}$ E) $\frac{\sqrt{2}}{3}$

25. The diagram shows a rectangle measuring 6×12 and a circle. The two shorter sides of the rectangle are tangents to the circle. The circle and rectangle have the same center. The region that lies inside both the rectangle and the circle is shaded. What is its area?



- A) $12\pi + 18\sqrt{3}$ B) $24\pi - 3\sqrt{3}$ C) $72 - 4\pi$
 D) $18\pi + 12\sqrt{3}$ E) $18\pi + 6\sqrt{3}$