Bergen County Academies Sunday Math Team

Mock AMC 8

Sunday, November 17th, 2013

$2013 = 3 \times 11 \times 61$

- d(N) is the number of positive divisors of n, including 1 and n itself.
- σ(N) is the **Sum of Divisors**. It equals the sum of all the positive divisors of n, including 1 and n itself.
- For a **Prime Number**, d(N)=2. The only divisors for a Prime Number are 1 and itself.
- The sum of the divisors of a **Deficient Number** is < than twice the sum of its proper divisors, $\sigma(N) < 2n$.
- The sum of the divisors of an **Abundant Number** is > than twice the sum of its divisors, $\sigma(N) > 2n$.
- The sum of the divisors of a **Perfect Number** =twice the number; that is, $\sigma(N) = 2n$.

N	Divisors of N	d(N)	σ(N)	Prime or Composite	Notes
2010	1, 2, 3, 5, 6, 10, 15, 30, 67, 134, 201, 335, 402, 670, 1005, 2010	16	4896	Composite	Abundant
2011	1, 2011	2	2012	Prime	Deficient
2012	1, 2, 4, 503, 1006, 2012	6	3528	Composite	Deficient
2013	1, 3, 11, 33, 61, 183, 671, 2013	8	2976	Composite	Deficient
2014	1, 2, 19, 38, 53, 106, 1007, 2014	8	3240	Composite	Deficient
2015	1, 5, 13, 31, 65, 155, 403, 2015	8	2688	Composite	Deficient
2016	1, 2, 3, 4, 6, 7, 8, 9, 12, 14, 16, 18, 21, 24, 28, 32, 36, 42, 48, 56, 63, 72, 84, 96, 112, 126, 144, 168, 224, 252, 288, 336, 504, 672, 1008, 2016	36	6552	Composite	Abundant
2017	1, 2017	2	2018	Prime	Deficient
2018	1, 2, 1009, 2018	4	3030	Composite	Deficient
2019	1, 3, 673, 2019	4	2696	Composite	Deficient
2020	1, 2, 4, 5, 10, 20, 101, 202, 404, 505, 1010, 2020	12	4284	Composite	Abundant
2021	1, 43, 47, 2021	4	2112	Composite	Deficient

Have Fun!

- Daniel Plotnick

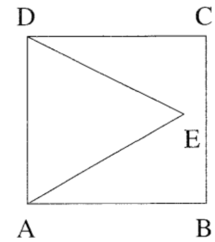
1. Find the sum of the digits of the product ($111111111 \; ... \; 111$) $\times$ 2013 where there are 2013 1's in the first number of the product.

A) 2013 B) 4026 C) 6039 D) 10065 E) 12078

2. Find the value of the expression $\dfrac{\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\cdots\left(1-\frac{1}{2013^2}\right)}{\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\cdots\left(1-\frac{1}{2013}\right)}$

A) $\frac{2013}{2}$ B) 1007 C) $\frac{2013}{1007}$ D) 1006 E) 671

3. The diagram shows an equilateral triangle ADE inside a square ABCD. What is the value of $\frac{area\ of\ \Delta\ ADE}{area\ of\ \Delta\ DEC}$?



A) $\frac{\sqrt{3}}{4}$ B) 1 C) $\frac{\sqrt{3}}{2}$
D) $\sqrt{3}$ E) 2

4. Circular coins of the same size are arranged on a very large table (the infinite plane) such that each coin touches six other coins. Find the percentage of the plane that is covered by the coins.

A) $\frac{49\sqrt{3}}{3}\pi\%$ B) $\frac{50\sqrt{3}}{3}\pi\%$ C) $16\sqrt{3}\pi\%$ D) $17\sqrt{3}\,\pi\%$ E) $18\sqrt{3}\pi\%$

5. Among the five real numbers below, which one is the smallest?

A) $\sqrt[2012]{2013}$ B) $\sqrt[2013]{2012}$ C) 2013 D) $\frac{2013}{2012}$ E) $\frac{2012}{2013}$

6. Among the five integers below, which one is the largest?

A) 2012^{2013} B) 20122013^{671} C) 2013^{2012}

D) $3^{\left(3^{\left(3^{3}\right)}\right)}$ E) $2^{2012} + 3^{2012} + \cdots + 2013^{2012}$

7. Among the following four statements on real numbers below, how many of them are correct?
"If $a < b$ and $a, b \neq 0$ then $\frac{1}{b} < \frac{1}{a}$"; "If $a < b$ then $ac < bc$";
"If $a < b$ then $a + c < b + c$"; "If $a^2 < b^2$ then $a < b$".

A) 0 B) 1 C) 2 D) 3 E) 4

8. What is the largest integer less than or equal to $\sqrt[3]{(2013)^3 + 3 \times (2013)^2 + 4 \times 2013 + 1}$

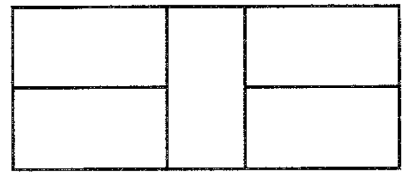
A) 2012 B) 2013 C) 2014 D) 2015 E) 2016

9. Evaluate the sum $\dfrac{2013}{1\times2} + \dfrac{2013}{2\times3} + \cdots + \dfrac{2013}{2012\times2013}$

A) 2012 B) 2013 C) 2014 D) 2015 E) 2016

10. Find the largest integer n such that $n^{6039} < 2013^{2013}$
A) 12 B) 13 C) 14 D) 15 E) 16

11. Five identical rectangles of area $8\ inches^2$ are arranged into a large rectangle as shown. Find the perimeter of the rectangle.



A) 14 B) 28 C) 48
D) 56 E) Cannot be determined.

12. Wen has two quarters (each 25 cents) and three half-dollars (each 50 cents) in his pocket. He takes two coins out of his pocket, at random, one after the other without replacement. What is the probability that the total value of the two coins taken out is 75 cents?

A) $\frac{16}{25}$ B) $\frac{9}{25}$ C) $\frac{12}{25}$ D) $\frac{3}{5}$ E) $\frac{13}{25}$

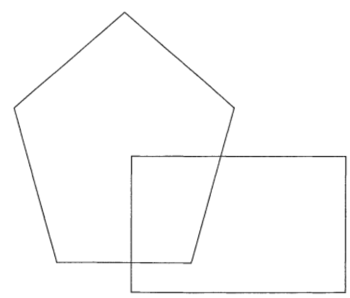
13. Find the integer part of $\dfrac{1}{\frac{1}{2007}+\frac{1}{2008}+\frac{1}{2009}+\frac{1}{2010}+\frac{1}{2011}+\frac{1}{2012}+\frac{1}{2013}}$

A) 1 B) 284 C) 285 D) 286 E) 2013

14. The number of ways to arrange 5 boys and 6 girls in a row such that girls can be next to other girls but boys cannot be next to other boys is $6! \times N$. Find N.

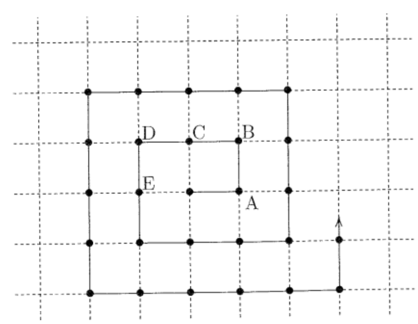
A) 5040 B) 2520 C) 1680 D) 1320 E) 1260

15. The diagram to the right shows a pentagon (made up of two regions) and a rectangle (also made up of two regions) with one region in common. The overlapped region is $\frac{3}{16}$ the area of the pentagon and $\frac{2}{9}$ the area of the rectangle. If the ratio of non-overlapping region of the pentagon to non-overlapping region of the rectangle is $\frac{m}{n}$ in lowest terms, find the value of $m+n$.



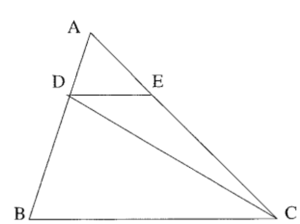
A) 47 B) 45 C) 30 D) 15 E) 7

16. You walk a spiraling maze on the Cartesian plane as follows: starting at the origin (0,0) and the first five stops are at A (1,0), B (1,1), C (0,1), D (-1,1), and E (-1,0). Your ninth stop is at the point (2,-1) and so on (see the diagram to the right.) What is the x-coordinate of the point which you would arrive at on your 2013[th] stop?



A) 9 B) 10 C) 11
D) 12 E) 13

17. In figure to the right, if DE is parallel to BC and the area of $\Delta ADE = 1$ and the area of $\Delta ADC = 4$ find the area of ΔDBC.



A) 9 B) 12 C) 15
D) 16 E) 18

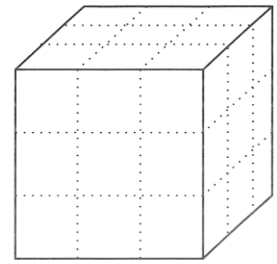
18. Let C be a circle with radius 2013. Suppose N points are placed inside the circle and the distance between any two points exceeds 2013. What is the largest possible value of N?

A) 5 B) 6 C) 671 D) 1006 E) 2013

19. Given that N is a positive integer and $S = 1 + 2 + 3 + \cdots + N$. The units digit of S cannot be some numbers. Find the sum of these numbers.

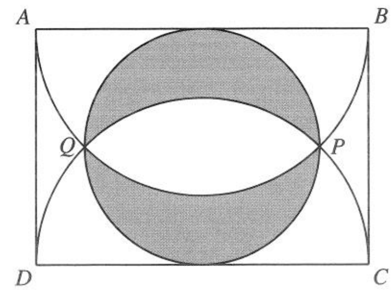
A) 13 B) 15 C) 18 D) 22 E) 30

20. A carpenter wishes to cut a wooden 3 x 3 x 3 cube into 27 1 x 1 x 1 cubes. She can do this easily by making 6 cuts through the cube, keeping the pieces together in the cube shape as shown. What is the minimum number of cuts needed if she is allowed to rearrange the pieces after each cut?



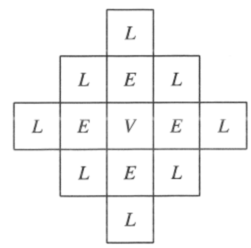
A) 2 B) 3 C) 4
D) 5 E) 6

21. Let ABCD be a rectangle with AB = 10. Draw semi-circles inside the rectangle with diameters AB and CD respectively. Let P and Q be the intersection points of the two semi-circles. If the circle with diameter PQ is tangent to AB and CD, then what is the area of the shaded region?



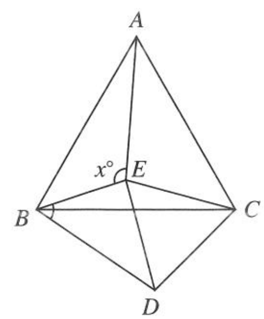
A) $\frac{25}{3}\pi$ B) $\frac{25}{2}\pi$ C) 25
D) $25\pi - \frac{25}{6}$ E) 25π

22. Starting from any of the L's, the word LEVEL can be spelled by moving either up, down, left or right to an adjacent letter. If the same letter may be used twice in a given spelling, how many different ways are there to spell the word LEVEL ?



A) 64 B) 72 C) 144
D) 160 E) 288

23. In the diagram, ΔABC and ΔCDE are equilateral triangles. Given that $\angle EBD = 62°$ and $\angle AEB = x°$, what is the value of x?



A) 100 B) 118 C) 120
D) 122 E) 131

24. There are four piles of stones: One with 6 stones, two with 8 stones, and one with 9 stones. Five players, numbered 1, 2, 3, 4 and 5 take turns, in the order of their numbers, starting with 1, choosing one of the piles and dividing it into two smaller piles. The loser is the player who can't do this. Which player loses?

A) 1 B) 2 C) 3 D) 4 E) 5

25. The last two digits of 9^{2013} are
A) 09 B) 29 C) 49 D) 69 E) 89