Bergen County Academies Sunday Math Team

Mock AMC 8

Sunday, November 3rd, 2013

$2013 = 3 \times 11 \times 61$

Find the next two years that also have exactly three distinct prime factors.

Find a formula for the volume of a pizza if the height (thickness) of the pizza is “a” and the radius of the pizza is “z”, use “pi” for π.

- Daniel Plotnick

1. What are the last two digits of $1 \times 2 \times 3 \times 4 \times \ldots \times 2011 \times 2012 \times 2013$?
   A) 13 B) 25 C) 00 D) 44 E) 50

2. What are the last two digits of $1 + 2 + 3 + 4 + \cdots + 2011 + 2012 + 2013$?
   A) 91 B) 78 C) 00 D) 51 E) 18

3. How many zeroes does the number $2013!$ end width. Remember $n!$ is defined as follows $n! = n \times (n-1) \times (n-2) \times \ldots \times 3 \times 2 \times 1.$
   A) 402 B) 482 C) 498 D) 501 E) 2013

4. If a polygon has its sum of interior angles smaller than 2013° , what is the maximum number of sides of the polygon
   A) 13 B) 14 C) 15 D) 16 E) 17

5. Using the digits 1, 2, 3, and 4 only once each to form a 4-digit number, how many of them are divisible by 11?
   A) 4 B) 5 C) 6 D) 7 E) 8

6. Suppose $a \neq 0, and\ b \neq 0\ and\ \frac{b}{a} = \frac{c}{b} = 2013.$ Find the value of $\frac{b+c}{a+b}$.
   A) 1 B) $\frac{2013}{2}$ C) 671 D) 2013 E) $\frac{1}{3}$

7. Suppose the polygon ABCDEF is a regular hexagon. What is the value of the quotient
$$\frac{Area\ of\ hexagon\ ABCDEF}{Area\ of\ triangle\ ACD}$$
   A) 2 B) 3 C) 4 D) 6 E) 12

8. Let p be a prime number such that the next larger number is a perfect square. Find the sum of all such prime numbers.
   A) 3 B) 18 C) 38 D) 47 E) 101

9. Find the number of even digits in the product of the two 10-digit numbers $2222222222 \times 9999999999.$
   A) 9 B) 21 C) 12 D) 11 E) 10

10. Which of the following is a possible number of diagonals of a convex polygon?
    A) 32 B) 45 C) 54 D) 63 E) 126

11. If $\frac{2013}{23} = a + \frac{1}{b + \frac{1}{c + \frac{1}{d}}}$, where a, b, c, and d are positive integers, find the value of a+b+c+d.
    A) 95 B) 96 C) 98 D) 99 E) 100

12. What is the largest positive integer n satisfying $n^{4026} < 5^{6039}$?

A) 9 B) 10 C) 11 D) 12 E) 13

13. The "4" button on my calculator is broken, so I cannot enter numbers which contain the digit 4. Moreover, my calculator does not display the digit 4 if 4 is part of an answer either. Thus, I cannot enter the calculation 2 x 14 and do not attempt to do so. And for example, the result of multiplying 3 by 18 is displayed as 5 instead of 54 and the result of multiplying 7 by 7 is displayed as 9 instead of 49. If I multiply a positive one-digit number by a positive two-digit number on my calculator and it displays 26, how many possibilities could I have multiplied. Order doesn't matter , so for example 2 x 13 and 13 x 2 count as one possibility.

A) 1 B) 2 C) 3 D) 6 E) 12

14. Given that $x > 0 \; and \; x - \frac{1}{x} = 5, \; find \; the \; value \; of \; \left(x + \frac{1}{x}\right)^2$.

A) 31 B) 29 C) 26 D) 25 E) None of these

15. Among the four statements on integers listed below, how many are always true?

"$If \; a < b \; then \; a^2 < b^2$ "; "$a^2 > 0 \; is \; always \; true$ ";

"$-a < 0 \; is \; always \; true$ "; "$If \; ac^2 < bc^2 \; then \; a < b$ ";

A) 4 B) 3 C) 2 D) 1 E) 0

16. Which of the following numbers is odd for any integer values of k?

A) $2013 + k^3$ B) $2013 + 7k$ C) $2013 + 2k^2$

D) $2013 + 2013k$ E) $2013k$

17. An unbiased six-sided dice is numbered 1 to 6. The dice is thrown twice and the two numbers added together for a score. Which of the following events has the highest probability of occurrence?

A) The total score is a multiple of 4. B) The total score is a prime number.
C) The total score is a perfect square. D) The total score is 7
E) The total score is a factor of 12.

18. 2013 students were interviewed. Of these, 1988 liked swimming and 1989 liked soccer. Find the greatest possible number of students who neither liked swimming nor soccer.

A) 9 B) 12 C) 24 D) 27 E) 36

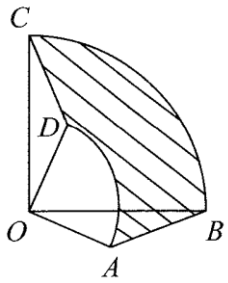
19. A brand of energy drink is available in shop A and shop B at an original price of $2.00 per bottle. Shop A provides a "buy 4 get 1 free" promotion and shop B provides a 15% discount if one buys 4 bottles or more. Find the minimum cost (in cents) if one wants to buy 13 bottles of energy drink. (You may buy bottles in either shop, or both shops.)

A) 2013 B) 2100 C) 2160 D) 2200 E) 2210

20. Let C_1 and C_2 be distinct circles of radius 7 inches that in the same plane and tangent to each other. Find the number of circles of radius 26 inches in this plane that are tangent to both C_1 and C_2.

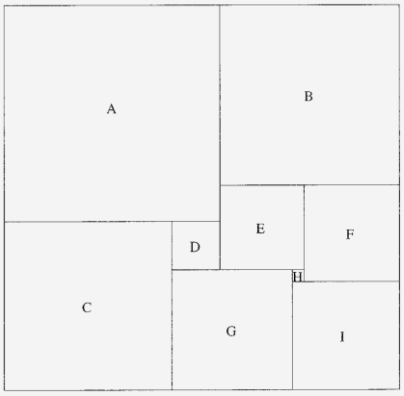
A) 12 B) 10 C) 8 D) 6 E) 4

21. In the diagram below, the radius of quadrant OAD is 4 and the radius of quadrant OBC is 8. Given that $\angle COD = 30°$, find the area of the shaded region ABCD.



A) 12π B) 13π C) 15π
D) 16π E) None of these

22. The rectangle shown is formed with nine squares of different sizes. The squares are labeled A, B, C, D, E, F, G, H, and I. Suppose that the side of square E is of length 7 inches Let the area of the rectangle be $x\ inches^2$.

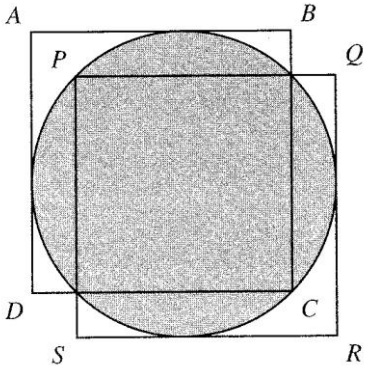


Find the value of x.

A) 992 B) 1056 C) 1088

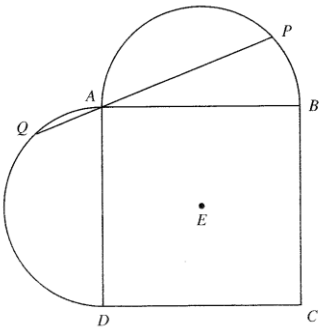
D) 1122 E) 1190

23. The diagram shows two identical squares, ABCD and PQRS, overlapping each other in such a way that their edges are parallel, and a circle of radius $\left(2 - \sqrt{2}\right)$ in. covered within these squares. Find the length of the side of square ABCD.



A) 1
B) $\sqrt{2}$
C) $\frac{1+\sqrt{2}}{2}$
D) $\frac{3-\sqrt{2}}{2}$
E) $3(\sqrt{2}-1)$

24. In the diagram at right, ABCD is a square, and E is the center of the square ABCD. P is a point on a semi-circle with diameter AB. Q is a point on a semi-circle with diameter AD. Moreover, Q, A, and P are collinear (that is, they are on the same line). Suppose QA = 14 in., AP = 46 in., and AE = x in. Find the value of x.



A) $34\sqrt{2}$
B) $17\sqrt{2}$
C) $17 + 17\sqrt{2}$
D) 34
E) $\sqrt{34}$

25. The product $1 \times 2 \times 3 \times \ldots \times n\ is\ denoted\ by\ n!$ For example, $4! = 1 \times 2 \times 3 \times 4 = 24$. Let $N = 1! \times 2! \times 3! \times 4! \times 5! \times 6! \times 7! \times 8! \times 9!$. How many factors of N are perfect squares?

A) 336 B) 480 C) 672 D) 720 E) 1008