

BCA Academy Sunday Math Team

Mock AMC 8 – November 6th, 2011

Answer Key:

1. C
2. D
3. C
4. D
5. B
6. C
7. B
8. E
9. D
10. A
11. D
12. B
13. A
14. B
15. A
16. A
17. B
18. A
19. E
20. C
21. B
22. D
23. C
24. D
25. B

Most Common Incorrect Answer

1. E
2. C
3. B
4. C
5. A
6. E
7. E
8. A
9. E
10. mix
11. E
12. E
13. B
14. D
15. mix
16. mix
17. C
18. mix
19. C
20. A
21. A
22. E
23. D
24. A
25. E

1. How much is $12 + 23 + 34 + 45 + 56 + 67 + 78 + 89$?

- A) 389 B) 396 **C) 404** D) 405 E) None of these

Just adding up the numbers is perfectly reasonable, and you will get 404. You might also notice that you can speed things up by organizing the addition as

$$(12 + 89) + (23 + 78) + (34 + 67) + (45 + 56) = (101) + (101) + (101) + (101) = 4 \times 101 = 404.$$

or you could organize the addition as

$$10 + 20 + \dots + 80 + 2 + 3 + \dots + 9 \text{ and then use your formula } 1 + \dots + N = N(N+1)/2$$

$$10 + 20 + \dots + 80 = 10(8 \times 9/2) = 360$$

$$2 + 3 + \dots + 9 = (9 \times 10)/2 - 1 = 45 - 1 = 44, \text{ so the answer is } 360 + 44 = 404$$

Yet another thing to notice is that the last digit of the sum has to be 4, but you would still have to check if it was none of these.

2. The closest integer to $\frac{2011}{2+0+1+1}$ is

- A) 500 B) 501 C) 502 **D) 503** E) 504

Just divide $2011/2+0+1+1 = 2011/4 = 502.75$, the closest integer is 503. "Closest" means rounding, so we would round 502.75 up to 503.

3. What is the remainder when dividing the sum $2001 + 2002 + 2003 + \dots + 2011$ by 2011?

- A) 0 B) 55 **C) 1956** D) 2011 E) 66

This question is about organizing your arithmetic:

You will end up dividing 22066 by 2011 and finding the remainder.

$$2001 + 2002 + \dots + 2011 = 11 \times 2000 + 1 + 2 + \dots + 11 = 11 \times 2000 + 11 \times 12/2 =$$

$$22000 + 66 = 22066. \text{ Dividing by 2011 is 10 remainder 1956.}$$

$$2011 \times 10 = 20110, \text{ and } 22066 - 20110 = 1956.$$

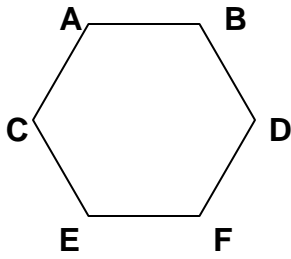
Another way to look at it is with "negative" remainders, you can view the remainders of each number 2001, ..., 2011 when divided by 2011 as -10, -9, -8, ..., -1, 0. The sum of these remainders is $-10 \times 11/2 = -55$, and $2011 - 55 = 1956$.

The most common wrong answer was B, which would be the answer if you were dividing by 2000 instead of 2011.

A lot of the answers were A, since you all know that $1 + \dots + N = N(N+1)/2$ is divisible by N, but that is not the situation here – be careful.

4. Esther draws the six vertices of a regular hexagon and then connects some of the points with lines to obtain a geometric figure. Which of the following shapes can she NOT draw

- A) right triangle B) kite C) obtuse triangle **D) square** E) trapezoid



So, let's draw a regular hexagon, and label the points A, B, C, D, E and F as shown in the figure. Remember, regular means all sides and angles are equal, so the interior angles will be 120. So, let's eliminate choices:

If I connect ABCD, I get a trapezoid, so choice E is out. If I connect ABD, this is a triangle with a 120 angle, which is obtuse, so choice C is out. Now, if I connect ADEC, I get a "kite" shape, so choice B is out. That leaves choices A and E. Let's connect BFE, now this is a triangle, and angle DFE was 120, but triangle BDF is isosceles since $BD = BF$, so angle $BFD = 30$, which makes angle BFE a 90 degree angle, so choice A is out. That leaves choice D, you can't draw a square. If you note, if you connect ABFE for example, you get a rectangle, not a square.

5. The product of four distinct natural numbers is 100. What is their sum?

A) 14 **B) 18** C) 20 D) 29 E) 103

Although 100 can be factored into 4 numbers in several ways $100 = 2 \times 2 \times 5 \times 5$, $1 \times 2 \times 2 \times 25$, ... there is only one way that has 4 distinct – "distinct" means different. $100 = 1 \times 2 \times 5 \times 10$, and $1 + 2 + 5 + 10 = 18$.

6. In a class there are 9 boys and 13 girls. Half of the children in the class have got a cold. How many girls at least have a cold?

A) 0 B) 1 **C) 2** D) 3 E) 4

There are $9 + 13 = 22$ kids in the class, so 11 of them have a cold. If all the boys had colds that would still leave 2 people with a cold, so at least 2 girls must have a cold. There could be more girls with a cold, but then there would be fewer boys with a cold.

7. 60 Math Team members can eat 6 boxes of Welch's Fruit Snacks in 6 minutes. How many math team members will it take to eat 100 boxes of Welch's Fruit Snacks in 100 minutes?

A) 100 **B) 60** C) 6 D) 10 E) 600

This is called a "work problem." The way to do these problems is to get things down to "unit consumption, "unit time," or "unit workers." In this case, we can see that the team eats 6 boxes in 6 minutes, so dividing by 6, they (all together) eat 1 box in 1 minute, so it will take them 100 minutes to eat 100 boxes. So, we are done since that's what the problem asks. So the 60 member team can eat the 100 boxes in 100 minutes.

8. Two sides of a triangle are 120 and 130 inches long. Which of the following numbers below could not be the length of the third side of the triangle?

A) 40 B) 99 C) 100 D) 150 **E) 260**

This is about the property of the sides of a triangle called the “triangle inequality.” A very important concept is that the sum of any two sides of a triangle must be bigger than the third side. So, if two sides are 120 and 130, the 260 length is a problem, it is longer than the sum of 120 and 130, so it can’t be the third side of this triangle.

9. In triangle ABC, angle C is three times bigger than angle A, and angle B is two times bigger than angle A. Then we can say that triangle ABC is

- A) Equilateral B) Isosceles C) Obtuse **D) Right** E) Acute

Let’s use a little algebra. Let’s let A be the measure of angle A, B be the measure of angle B, and C be the measure of angle C. Then $C = 3A$, and $B = 2A$.

We know that the angles of a triangle add up to 180, so $A + B + C = 180$, now substituting, gives $A + 2A + 3A = 180$, or $6A = 180$, so $A = 30$, and $B = 60$, and $C = 90$. So it is a 30-60-90 right triangle.

10. Nina has \$147 and Rebecca has \$57. How much money does Nina need to give Rebecca so that she would have twice as much money left as Rebecca has then?

- A) \$11** B) \$19 C) \$30 D) \$45 E) \$49

Again, let’s use a little algebra, suppose Nina gives Rebecca D dollars, then

$$147 - D = 2 \times (57 + D)$$

$$147 - D = 114 + 2D$$

$$147 = 114 + 3D$$

$$33 = 3D$$

$$11 = D$$

11. Points A, B, C and D are marked on the straight line in some order. It is known that AB = 13, BC = 11, CD = 14 and DA = 12. What is the distance between the farthest two points?

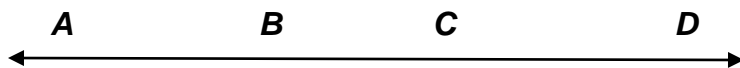
- A) 14 B) 38 C) 50 **D) 25** E) None of these

The figure is as follows, with $DA = 12$, $AC = 3$, and $CB = 10$.

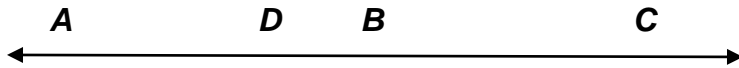


Note that the problem gave a gratuitous “hint” by referring to the “AD” distance as DA, implying that D was to the left of A, although you can’t rely on this in general.

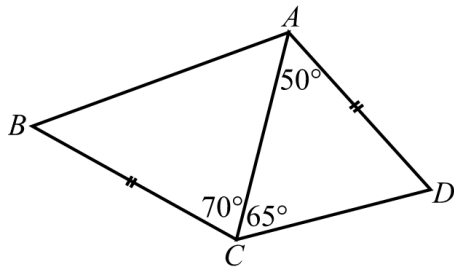
To figure this out this diagram, just start laying it out left to right and see what happens, if you put the segments as



This doesn’t work since AD would be 38 here, but it is supposed to be 12, so D can’t be to the left of C. Now try it with CD going towards the left:



This doesn't work either since AD would be 11 here, and it is 12. Now you know that C has to be to the left of B. Now check where D can be again, and so on, until you find the right configuration.



12. In the figure at left, in quadrilateral ABCD, we have $AD = BC$, $\angle DAC = 50^\circ$, $\angle DCA = 65^\circ$, $\angle ACB = 70^\circ$. Find the value of $\angle ABC$.

- A) 50° **B) 55°** C) 60° D) 65°
 E) impossible to determine

In triangle ACD, the angles add up to 180, so $50 + 65 + \angle ADC = 180$, and $\angle ADC$ is 65. So triangle ACD is isosceles, with $AD = AC$, but $AD = BC$ from the problem, so $BC = AC$, and then triangle CBA is also isosceles with $AC = BC$, so in triangle ABC, the base angles $\angle ABC$ and $\angle BCA$ are equal. Their sum is 110, so they must each be 55, and so angle $ABC = 55$.

13. A boy always speaks the truth on Thursdays and Fridays, always tells lies on Tuesdays, and randomly tells the truth or lies on other days of the week. On seven consecutive days he was asked what his name was, and on the first six days he gave the following answers in order: Kelvin, Alex, Kelvin, Alex, Tony, Alex. What did he answer on the seventh day?

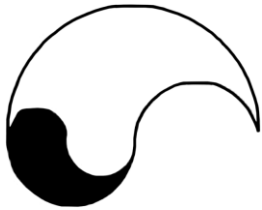
- A) Kelvin** B) Alex C) Tony D) Ryan E) impossible to determine

This is a tricky logic problem. Since he tells the truth on Thursdays and Fridays, his answer has to be the same true one on both of those days, but the list has alternating answers. That means that none of first five days' answers listed could have been on a Thursday. That leaves two possibilities, either the last answer listed, "Alex," was on a Thursday, so he also answered "Alex" on the seventh day, or the last answer "Alex," was on a Wednesday, and the seventh day unlisted answer was on a Thursday and he would have answered "Kelvin," since then the first "Kelvin" listed would then have been on a Friday, and they would both have to be true. Now in the first case, his Tuesday answer would then be "Alex," but he always lies on Tuesday, so it can't be this case. That leave answering Kelvin, and his Tuesday answer in this case was "Tony," which is a lie.

Here is a table with the two possibilities:

Saturday	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday
Kelvin	Alex	Kelvin	Alex	Tony	Alex	

Friday	Saturday	Sunday	Monday	Tuesday	Wednesday	Thursday
Kelvin	Alex	Kelvin	Alex	Tony	Alex	



14. The logo shown at left, is made entirely from semicircular arcs of radius 2 cm, 4 cm or 8 cm. What fraction of the logo is shaded?

- A) $1/3$ **B) $1/4$** C) $1/5$ D) $1/6$ E) $1/7$

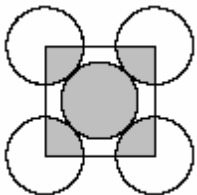
The entire logo is made up of an 8 cm semi-circle with a 4 cm semi-circle cut-out and a 4 cm semi-circle addition (made up of both shaded and un-shaded parts). So the area of the ENTIRE logo is just $\frac{1}{2} \pi 8^2 = \frac{1}{2} 64\pi = 32\pi$, since the semi-circular part that is left out equals the semi-circular part that is added in area. Now the shaded region is made up of 4 cm semi-circle with a 2 cm semi-circle un-shaded part and an addition of a 2 cm shaded semi-circle, and once again, the shaded addition cancels out with the un-shaded removed part. The area of the shaded region is the $\frac{1}{2} \pi 4^2 = \frac{1}{2} 16\pi = 8\pi$. So the area of the shaded region to the ENTIRE region is $8\pi/32\pi = 1/4$.

Note: Many people answered A, $1/3$, which is the ratio of the shaded region to the un-shaded region, which would be a perfectly legitimate alternate question to pose, and many people answered D, I guess since it “looks” like about a 6^{th} , don’t let appearances deceive!

15. What percent of all natural numbers from 1 to 10000 are perfect squares?

- A) 1%** B) 1.5% C) 2% D) 2.5% E) 5%

Note that $10000 = 100^2$ and of course $1^2 = 1$, so there are 100 squares from 1 to 10000, and the percentage of square would be $100/10000 = .01$, or 1%.



16. In the diagram, the five circles have the same radii, and they touch as shown. The small square joins the centers of the four outer circles. The ratio of the shaded area of all the circles to the non-shaded area of all the circles is:

- A) 2:3** B) 1:3 C) 2:5 D) 5:4 E) 1:4

Since the five circles all have the same radii, and the vertices of the square are the centers of the four outer circles, we know that the shaded part of each outer circle is $\frac{1}{4}$ of the circle, since the square has interior angle 90 , which is $\frac{1}{4}$ of 360 . So we have four quarters of a circle and one full circle shaded, which makes 2 circles shaded. The non-shaded area is $4 \times \frac{3}{4}$ circles by the same reasoning, or equivalent to 3 circles. The ratio of the shaded part to the non-shaded part is then $2 : 3$.

Be careful, more than a few chose “C” which is the ratio of the shaded part to all the circles area, not what was asked in the question.

17. The people of planet Zork call themselves Zugwugs and they have arithmetic very similar to ours. They have parenthesis, addition, subtraction, multiplication and division. However, their order of operations are different. On Zork, parenthesis are evaluated first, followed by addition and subtraction, then multiplication and then finally division, and from right to left. They remember this with the mnemonic “PASMDE,” which in Zugwug language is an acronym for

Puk Ark Sok Mog Dig Eeg and translates as "Please excuse my dear aunt Sally." Evaluate the following expression using Zork arithmetic.

$$(13 + 16 \div 2 - 1) - (5 + 7 \times 3 - 20)$$

- A) 14 **B) 233** C) 224 D) 135 E) 23

The simplification sequence using Zork arithmetic is as follows

$$\begin{aligned} &(13 + 16 \div 2 - 1) - (5 + 7 \times 3 - 20) \\ &(13 + 16 \div 2 - 1) - (5 + 7 \times -17) \\ &(13 + 16 \div 2 - 1) - (12 \times -17) \\ &(13 + 16 \div 2 - 1) - (12 \times -17) \\ &(13 + 16 \div 2 - 1) - (-204) \\ &(13 + 16 \div 1) - (-204) \\ &(29 \div 1) - (-204) \\ &29 - -204 \\ &233 \end{aligned}$$

There are some problems with the wording of this question. Are you supposed to also do the binary operations from right to left as well? In this case, the simplification sequence would be as follows:

$$\begin{aligned} &(13 + 16 \div 2 - 1) - (5 + 7 \times 3 - 20) \\ &(13 + 16 \div 2 - 1) - (5 + 7 \times 17) \\ &(13 + 16 \div 2 - 1) - (12 \times 17) \\ &(13 + 16 \div 2 - 1) - (12 \times 17) \\ &(13 + 16 \div 2 - 1) - (204) \\ &(13 + 16 \div -1) - (-204) \\ &(29 \div -1) - (-204) \\ &-1/29 + 204 \\ &203 \frac{28}{29} \end{aligned}$$

but 203 28/29 is not a choice, so that is not the interpretation. The problem only says the order of operations is right to left, not the binary operations themselves.

Are number also read right to left? What about that interpretation. Well, in that case you would have the number "02" from the 20, but even on Zork, they don't use a 0 as a lead digit for numbers.

18. Which of the following fractions has the largest value?

- A) 7/8** B) 66/77 C) 555/666 D) 4444/5555 E) 33333/44444

In looking at the fractions, the last 4 can easily be reduced by dividing by a common factor. B's common factor is 11, C's common factor is 111, D's common factor is 1111, and E's common factor is 11111, and the fractions reduce to B) 6/7, C) 5/6, D) 4/5, and E) 3/4.

At this point, you could simply convert to decimal and see that 7/8 is the largest.

However, let's look at this a little bit and let's invert (take the reciprocal) of them all, then

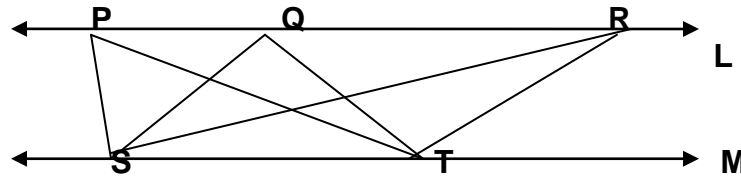
$A = 8/7 = 1 + 1/7$, $B = 7/6 = 1 + 1/6$, $C = 6/5 = 1 + 1/5$, $D = 5/4 = 1 + 1/4$, and $E = 4/3 = 1 + 1/3$

Whichever one was biggest would now be smallest. If we now subtract 1 from each that would not change anything about the ordering of the values, then

$A = 1/7$, $B = 1/6$, $C = 1/5$, $D = 1/4$, and $E = 1/3$

So, the value of A is now the smallest, but since we inverted the fractions, the original would be the largest.

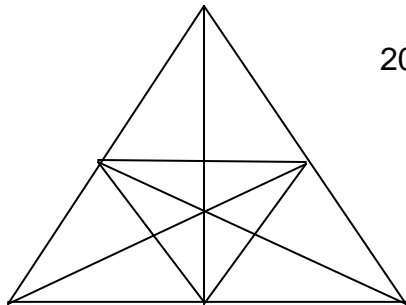
The point is it is easier to see the relationship with the inverted and reduced fractions. This comes up if the values in the choices don't have such clean common factors.



19. In the above figure, lines L and M are parallel. Points S and T are on line M and points P, Q and R are on line L. Three triangles are drawn, ΔPST , ΔQST , and ΔRST . Let the area of ΔPST be equal to a, the area of ΔQST be equal to b, and the area of ΔRST be equal to c. Which of the following inequalities is true?

- A) $a > b > c$ B) $a > c > b$ C) $c > a > b$ D) $c > b > a$ **E) None of these**

This is sort of a “trick” question, but the idea is to see if you are confident in your area formula. The area of a triangle is $\frac{1}{2}$ base \times height. Note that the base ST of all three triangle is the same. Also, the height of each triangle is the same since lines L and M are parallel. The “height” or altitude of a triangle can be measured “outside” the triangle. All three triangles have the same equal area $a = b = c$.



20. How many triangles are in the figure?

- A) 24 B) 25 **C) 47** D) 48 E) 72

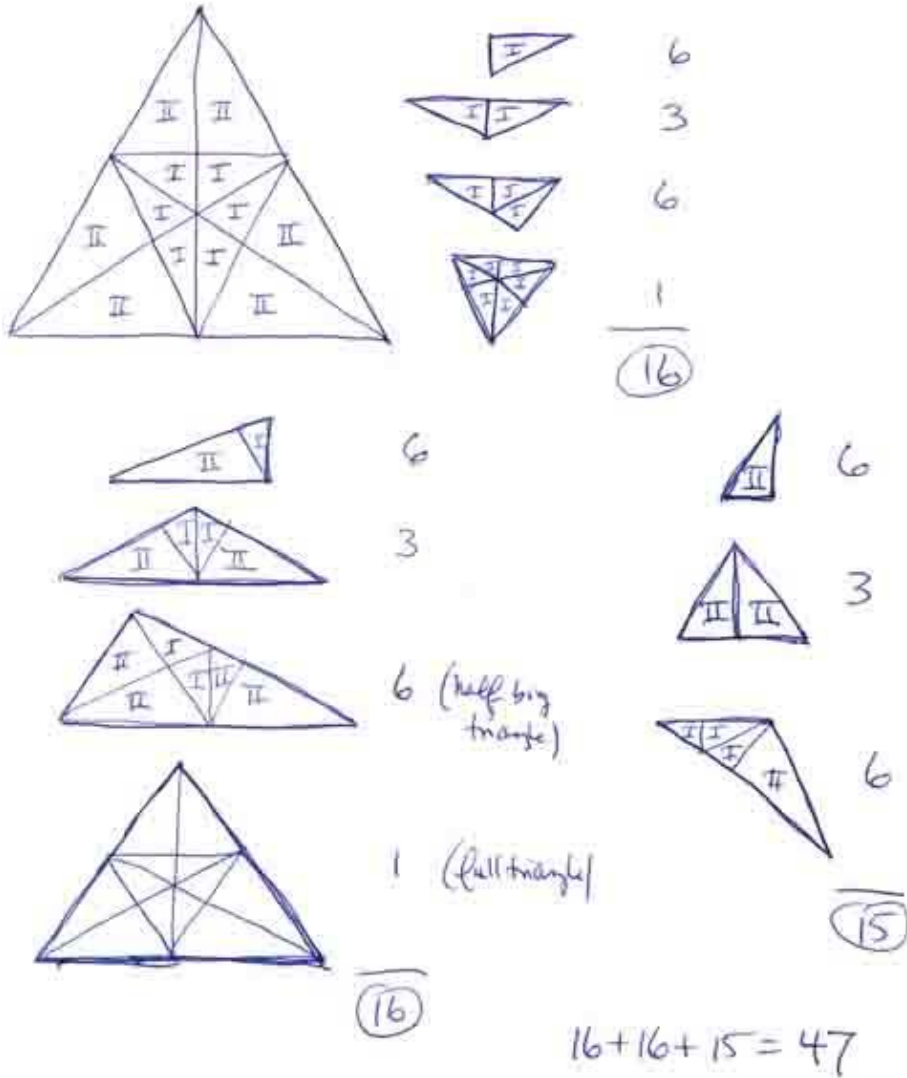
This question is a time waster. There are often time wasters on an AMC 8. Be careful not to get trapped.

So, anyway, how do we do it? You have to use organized counting. The best way is as follows: Label the smaller right triangles as type “I” and the larger right triangles as type “II,” and now list each possible triangle and an associated count, as in the drawing.

It is possible to speed up the count if you notice that the number of triangles in the “smaller” picture – the one made of all type “I” triangles is going to be the same as the number of triangles in the “larger” triangle with the “smaller” one “removed – the larger one is then

made up of 6 triangles of type "I+II" together, then you know the total is at least 32, and you've eliminated two choices easily. This is the type of thing you notice in retrospect though.

The other three types of triangles in the figure still have to be counted to get the extra 15 triangles.



21. Two fractions are equally spaced between $\frac{1}{4}$ and $\frac{2}{3}$. The smaller of the two fractions is

- A) $\frac{13}{24}$ **B) $\frac{7}{18}$** C) $\frac{29}{36}$ D) $\frac{5}{12}$ E) $\frac{1}{3}$

This is a very important question involving a basic fraction concept. The answer is $\frac{1}{4} + \frac{1}{3}(\frac{2}{3} - \frac{1}{4})$ That is $\frac{1}{3}$ of the way from $\frac{1}{4}$ to $\frac{2}{3}$ added to where you started at $\frac{1}{4}$. This simplifies to $\frac{7}{18}$.

You should understand the general principle. If you want to be $\frac{p}{q}$ of the way from x and y , you want to calculate $x + \frac{p}{q}(y - x)$. If doesn't matter if the end points are fractions.

If you want to be $\frac{p}{q}$ of the way from $\frac{a}{b}$ and $\frac{c}{d}$, you want to calculate $\frac{a}{b} + \frac{p}{q}(\frac{c}{d} - \frac{a}{b})$.

How does this result relate to the simpler problems of finding a number half-way between two numbers?

Normally, to find a number halfway between two number x and y , you just take $(x + y)/2$, note that this can be rewritten as $x + \frac{1}{2}(y-x)$ which is the more general concept. That is half the distance from x to y .

22. While playing around a creek, two boys, Wen and Arthur find an ordinary six-sided die buried in the dirt. Wen washes it off in the water and challenges Arthur to a contest. Each of the boys rolls the die exactly once. Wen's roll is 2 higher than Arthur's. "Let's play once more," says Arthur. Let $\frac{a}{b}$ be the probability that the difference between the outcomes of the two dice rolls is again exactly 2 (regardless of which roll is higher), where a and b are relatively prime positive integers. Find $a + b$.

- A) 23 B) 7 C) 5 **D) 11** E) 19

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

Always draw a nice table with dice rolls. As you see in the second table, we list the absolute difference of the two boys dice rolls. There are 8 out of 36 times do we get a difference of 2. So $\frac{8}{36} = \frac{2}{9} = \frac{a}{b}$, so $a = 2$, $b = 9$, and $a+b = 11$.

23. Nikhil writes down 11 1's in a row and randomly writes + or - between each pair of consecutive 1's. For example, $1 + 1 + 1 - 1 - 1 + 1 - 1 + 1 - 1 - 1 - 1$

What is the probability that the value of the expression Nikhil wrote down is 5?

The first number is 1 and doesn't change sign, so the 10 other ones have to add and subtract to a value of 4. So we have to put 10 +'s and -'s, and this can be done in $2^{10} = 1024$ ways. Now we have 10 1's that have to add up to four, and if you think about it, this only happens where we put 7 plus signs to give us +7, and 3 minus signs to give us -3, and $7 - 3 = 4$. So we have to put 3 minus signs in up to 10 places, which can be done in $10c3$ ways, or

$10!/7!3! = 10 \times 9 \times 8 / 3 \times 2 \times 1 = 120$. There are 2^{10} ways of writing down the +’s and –’s, so the answer is $120/2^{10} = 120/1024 = 15/128$.

- A) $1/8$ B) $3/16$ **C) $15/128$** D) $17/128$ E) $45/512$

24. Hannah wants to complete the missing digits of the number $2 _ _ 8$ with two different digits so that the resulting number is divisible by 9. In how many different ways could this be accomplished?

- A) 7 B) 8 C) 9 **D) 10** E) 11

The number is divisible by 9, so the sum of the digits must be divisible by 9, so let the two digits be a and b , then $2 + a + b + 8 = 10 + a + b$ must be divisible by 9. Since a and b are digits, and have values between $0 \leq a, b \leq 9$, $10 + a + b$ is between 10 and 29 and divisible by 9, so $10 + a + b$ has to be 18 or 27, and then $a + b$ has to be 8 or 17. In the first case (a,b) can be $(0,8)$, $(1,7)$, $(2,6)$, $(3,5)$, $(4,4)$, $(5,3)$, $(6,2)$, $(7,1)$, or $(8,0)$ which is 9 possibilities. If $a + b$ is 17, then there are only two possibilities $(8,9)$ and $(9,8)$. Now the problem said the digits a and b where different, so the $(4,4)$ case is out. So the total number of possibilities is $9 + 2 - 1 = 10$.

There are some problems with the wording of this question. For example, the “different” might mean to be different from 2 and 8, not from each other. In this case, the answer would be 5 possibilities (including $(4,4)$ in this interpretation), which is not one of the choices. This can happen on a contest. If your interpretation leads to an answer that is not a choice, reread the question to try and figure out the alternate interpretation.

25. Consider a cube with side length 4. Through one face I drill a circular hole of radius 1 all the way through to the opposite side, with the center of the circle also the center of the square side of the cube. Through another side I drill a square hole of side 2 all the way through to the opposite side, with the center of the square hole also the center of the square side of the cube, and the sides of the square are parallel to the sides of the cube. I then drill a third hole, shaped like an equilateral triangle with side $\sqrt{3}$ all the way through to the opposite side, with the center of the triangular hole also the center of the square of the cube. What is the surface area of the new solid I have created by drilling the holes (include the surface area from drilling the holes.)

- A) $120 + \pi + 9\sqrt{3}$ **B) $120 + 3\sqrt{3}$** C) $120 + 2\pi + 9\sqrt{3}$
D) $120 + \frac{9\sqrt{3}}{2}$ E) $120 + 2\pi + \frac{9\sqrt{3}}{2}$

The cube is side length 4, so we start with a surface area of $6 \times 4 \times 4 = 96$.

Now, drill the square hole first, we will lose surface area from the 2 squares of area 2×2 on each end of the hole, but we add surface area of the 4 interior rectangular surfaces of area 2×4 . So the surface area of the cube with just the square hole drilled is $96 - 2 \times 2 \times 2 + 4 \times 2 \times 4 = 96 - 8 + 32 = 120$.

Now, drill the circular hole, it doesn’t matter what side you are on. We will lose surface area from the 2 circles of area $\pi R^2 = \pi(1)^2 = \pi$ from each end, and we will add the surface area of the cylindrical interior of the hole. Since the part intersecting the square hole does not add to surface area (the circular hole fits entirely in the square hole,) this will be equal to the lateral surface area of 2 cylinders of height 1 and radius 1, each of which is $2\pi RH = 2\pi(1)(1) = 2\pi$. So we add new surface area of $2 \times 2\pi = 4\pi$.

And here’s where everyone goes wrong. The circular hole also intersects the sides of the square hole, so also removes additional surface area of two circular holes from the surface area of the interior rectangular surfaces of the square hole. We actual lost 4 circles of area $\pi(1)^2 = \pi$, not 2 circles worth of surface area.

So we gain 4π in surface area, and lost 4π in surface area. So, the surface area of the cube with the square hole drilled and the circular hole drilled is still 120!

At this point, you could guess choice B or D, but since the surface area is likely to increase with the triangular hole, you could go ahead with a choice B as a good guess.

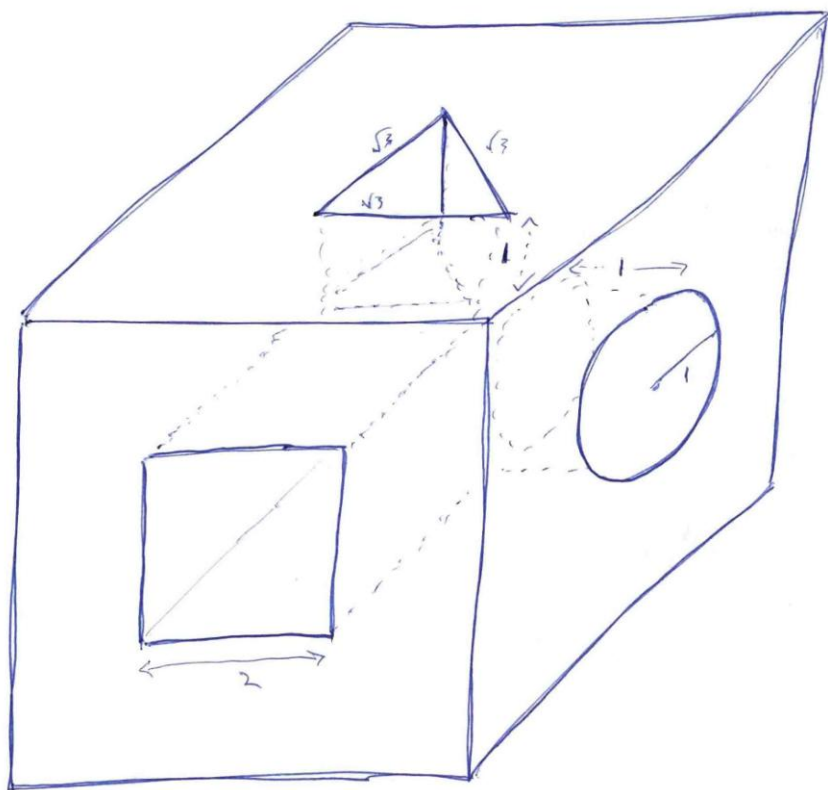
Now drill the triangular hole. The triangle has side $\sqrt{3}$ and we use the area formula for an equilateral triangle of side s is $(s^2\sqrt{3})/4$, so the area of the triangle in this case is $(3\sqrt{3})/4$. Once again our surface area will lose 4 of these triangular areas, not just 2, or $4 \times (3\sqrt{3})/4 = 3\sqrt{3}$.

And finally, how much do we add in surface area? Once again, the triangular hole fits entirely inside the square hole, so the additional surface area is gained by the rectangular sides of the triangular hole. There are 6 of these, each with sides 1 and $\sqrt{3}$, so we add an additional surface area of $6 \times 1 \times \sqrt{3} = 6\sqrt{3}$. So, we lose $3\sqrt{3}$ and gain $6\sqrt{3}$.

The final surface area is $120 - 3\sqrt{3} + 6\sqrt{3} = 120 + 3\sqrt{3}$. Choice B.

Note, choice E is what you get if you forget to account for the intersection of the circular hole and the triangular hole with the sides of the rectangular hole.

Trying to do it all at once is confusing. You can often do area, perimeter, volume and surface area questions incrementally. This can also allow you to eliminate choices as you go.



An important point is to make sure the circular hole would fit entirely in the square hole as it passes through it, and the same for the triangular hole. The radius of the circle is 1, and a circle of radius 1 can be inscribed in a square of side 2. For the triangle, it is less obvious, but you should be able to see that an equilateral triangle of side $\sqrt{3}$ would be inscribed in a circle of radius 1, and thus would also fit entirely within the square hole.