MAA AMC

American Mathematics Competitions

MAA American Mathematics Competitions

39th Annual

AMC 8

Thursday, January 18, 2024 through Wednesday, January 24, 2024

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR COMPETITION MANAGER TELLS YOU TO BEGIN.
2. This is a 25-question multiple-choice competition. For each question, only one answer choice is correct.
3. Mark your answer to each problem on the answer sheet with a #2 pencil. Check blackened answers for accuracy and erase errors completely. Only answers that are properly marked on the answer sheet will be scored.
4. SCORING: You will receive 1 point for each correct answer, 0 points for each problem left unanswered, and 0 points for each incorrect answer.
5. Only blank scratch paper, rulers, and erasers are allowed as aids. Prohibited materials include calculators, smartwatches, phones, computing devices, compasses, protractors, and graph paper. No problems on the competition will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. You will have 40 minutes to complete the competition once your competition manager tells you to begin.

The problems and solutions for this AMC 8 were prepared
by the MAA AMC 8 Editorial Board under the direction of:
Silva Chang and Steven Klee

1. What is the ones digit of

$$222{,}222 - 22{,}222 - 2{,}222 - 222 - 22 - 2\,?$$

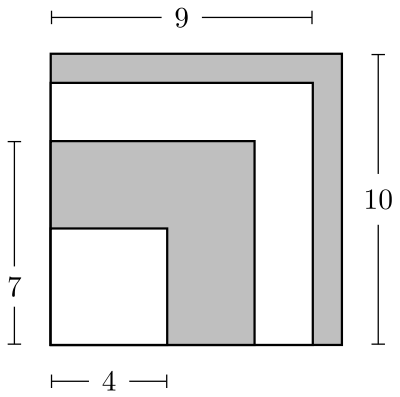
(A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

2. What is the value of this expression in decimal form?

$$\frac{44}{11} + \frac{110}{44} + \frac{44}{1100}$$

(A) 6.4 **(B)** 6.504 **(C)** 6.54 **(D)** 6.9 **(E)** 6.94

3. Four squares of side length 4, 7, 9, and 10 units are arranged in increasing size order so that their left edges and bottom edges align. The squares alternate in color white-gray-white-gray, respectively, as shown in the figure. What is the area of the visible gray region in square units?



(A) 42 **(B)** 45 **(C)** 49 **(D)** 50 **(E)** 52

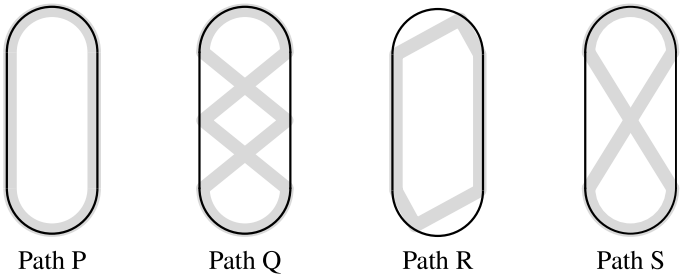
4. When Yunji added all the integers from 1 through 9, she mistakenly left out a number. Her incorrect sum turned out to be a square number. Which number did Yunji leave out?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

5. Aaliyah rolls two standard 6-sided dice. She notices that the product of the two numbers rolled is a multiple of 6. Which of the following integers *cannot* be the sum of the two numbers?

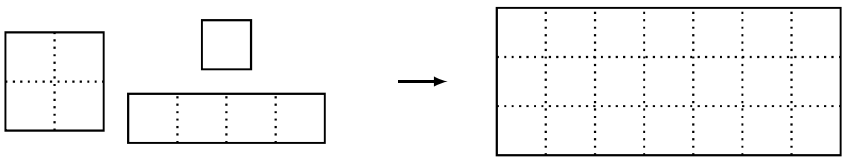
(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

6. Sergei skated around an ice rink, gliding along different paths. The gray lines in the figures below show four of the paths labeled P, Q, R, and S. What is the sorted order of the four paths from shortest to longest?



(A) P, Q, R, S **(B)** P, R, S, Q **(C)** Q, S, P, R **(D)** R, P, S, Q **(E)** R, S, P, Q

7. A 3×7 rectangle is covered without overlap by 3 shapes of tiles: 2×2, 1×4, and 1×1, shown below. What is the minimum possible number of 1×1 tiles used?



(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

8. On Monday Taye has \$2. Every day, he either gains \$3 or doubles the amount of money he had on the previous day. How many different dollar amounts could Taye have on Thursday, 3 days later?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

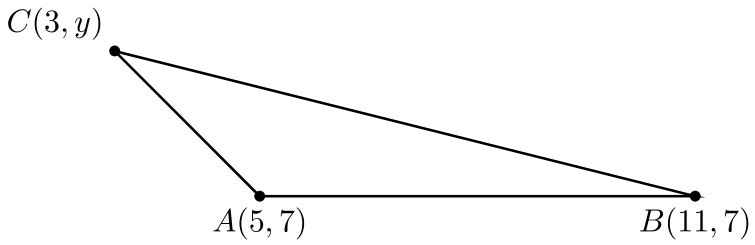
9. All of the marbles in Maria's collection are red, green, or blue. Maria has half as many red marbles as green marbles and twice as many blue marbles as green marbles. Which of the following could be the total number of marbles in Maria's collection?

(A) 24 **(B)** 25 **(C)** 26 **(D)** 27 **(E)** 28

10. In January 1980 the Mauna Loa Observatory recorded carbon dioxide (CO_2) levels of 338 ppm (parts per million). Over the years the average CO_2 reading has increased by about 1.515 ppm each year. What is the expected CO_2 level in ppm in January 2030? Round your answer to the nearest integer.

(A) 399 **(B)** 414 **(C)** 420 **(D)** 444 **(E)** 459

11. The coordinates of $\triangle ABC$ are $A(5,7)$, $B(11,7)$ and $C(3,y)$, with $y > 7$. The area of $\triangle ABC$ is 12. What is the value of y ?



(A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

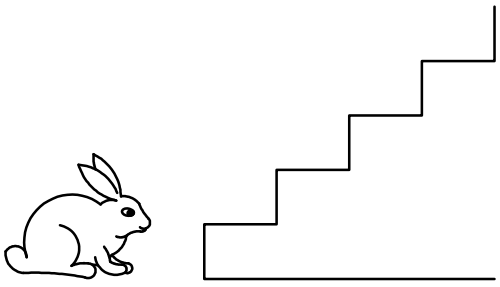
12. Rohan keeps a total of 90 guppies in 4 fish tanks.

- There is 1 more guppy in the 2nd tank than in the 1st tank.
- There are 2 more guppies in the 3rd tank than in the 2nd tank.
- There are 3 more guppies in the 4th tank than in the 3rd tank.

How many guppies are in the 4th tank?

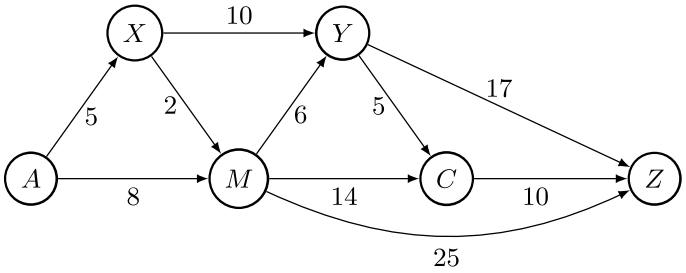
(A) 20 **(B)** 21 **(C)** 23 **(D)** 24 **(E)** 26

13. Buzz Bunny is hopping up and down a set of stairs, one step at a time. In how many ways can Buzz start on the ground, make a sequence of 6 hops, and end up back on the ground? (For example, one sequence of hops is up-up-down-down-up-down.)



(A) 4 **(B)** 5 **(C)** 6 **(D)** 8 **(E)** 12

14. The one-way routes connecting towns A, M, C, X, Y, and Z are shown in the figure below (not drawn to scale). The distances in kilometers along each route are marked. Traveling along these routes, what is the shortest distance from A to Z in kilometers?



(A) 28 **(B)** 29 **(C)** 30 **(D)** 31 **(E)** 32

15. Let the letters F, L, Y, B, U, G represent distinct digits. Suppose $\underline{F}\,\underline{L}\,\underline{Y}\,\underline{F}\,\underline{L}\,\underline{Y}$ is the greatest number that satisfies the equation

$$8 \cdot \underline{F}\,\underline{L}\,\underline{Y}\,\underline{F}\,\underline{L}\,\underline{Y} = \underline{B}\,\underline{U}\,\underline{G}\,\underline{B}\,\underline{U}\,\underline{G}\,.$$

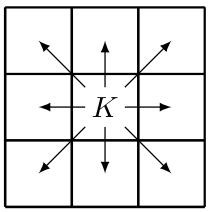
What is the value of $\underline{F}\,\underline{L}\,\underline{Y} + \underline{B}\,\underline{U}\,\underline{G}$?

(A) 1089 **(B)** 1098 **(C)** 1107 **(D)** 1116 **(E)** 1125

16. Minh enters the numbers 1 through 81 into the cells of a 9×9 grid in some order. She calculates the product of the numbers in each row and column. What is the least number of rows and columns that could have a product divisible by 3?

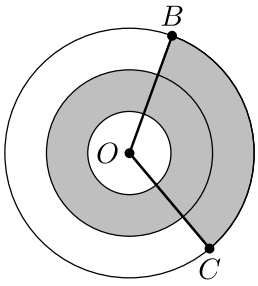
(A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

17. A chess king is said to *attack* all the squares one step away from it, horizontally, vertically, or diagonally. For instance, a king on the center square of a 3×3 grid attacks all 8 other squares, as shown below. Suppose a white king and a black king are placed on different squares of a 3×3 grid so that they do not attack each other. In how many ways can this be done?



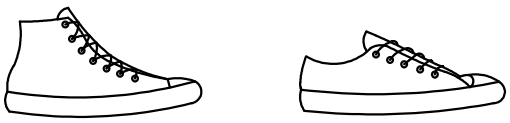
(A) 20 **(B)** 24 **(C)** 27 **(D)** 28 **(E)** 32

18. Three concentric circles centered at O have radii of 1, 2, and 3. Points B and C lie on the largest circle. The region between the two smaller circles is shaded, as is the portion of the region between the two larger circles bounded by central angle BOC, as shown in the figure below. Suppose the shaded and unshaded regions are equal in area. What is the measure of $\angle BOC$ in degrees?



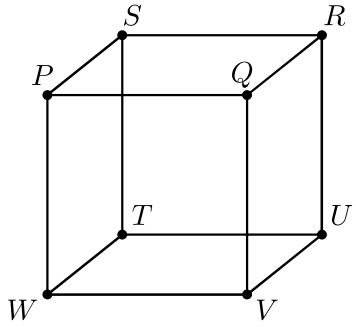
(A) 108 **(B)** 120 **(C)** 135 **(D)** 144 **(E)** 150

19. Jordan owns 15 pairs of sneakers. Three fifths of the pairs are red and the rest are white. Two thirds of the pairs are high-top and the rest are low-top. The red high-top sneakers make up a fraction of the collection. What is the least possible value of this fraction?



(A) 0 **(B)** $\frac{1}{5}$ **(C)** $\frac{4}{15}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{2}{5}$

20. Any three vertices of the cube $PQRSTUVW$, shown in the figure below, can be connected to form a triangle. (For example, vertices P, Q, and R can be connected to form isosceles $\triangle PQR$.) How many of these triangles are equilateral and contain P as a vertex?

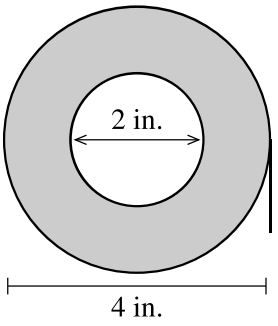


(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 6

21. A group of frogs (called an *army*) is living in a tree. A frog turns green when in the shade and turns yellow when in the sun. Initially the ratio of green to yellow frogs was $3 : 1$. Then 3 green frogs moved to the sunny side and 5 yellow frogs moved to the shady side. Now the ratio is $4 : 1$. What is the difference between the number of green frogs and yellow frogs now?

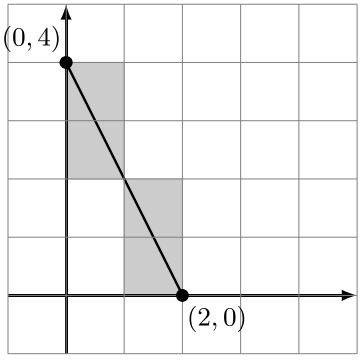
(A) 10 **(B)** 12 **(C)** 16 **(D)** 20 **(E)** 24

22. A roll of tape is 4 inches in diameter and is wrapped around a ring that is 2 inches in diameter. A cross section of the tape is shown in the figure below. The tape is 0.015 inches thick. If the tape is completely unrolled, approximately how long would it be? Round your answer to the nearest 100 inches.



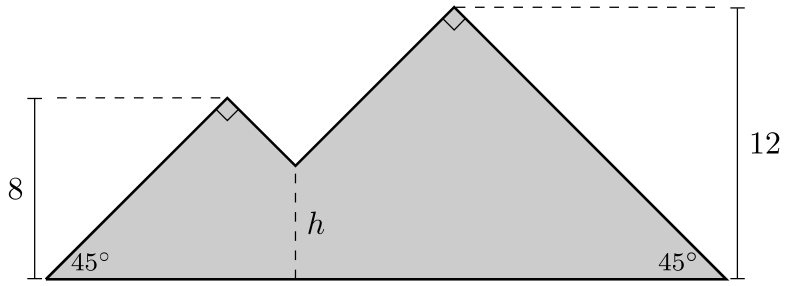
(A) 300 **(B)** 600 **(C)** 1200 **(D)** 1500 **(E)** 1800

23. Rodrigo has a very large piece of graph paper. First he draws a line segment connecting point $(0, 4)$ to point $(2, 0)$ and colors the 4 cells whose interiors intersect the segment, as shown below. Next Rodrigo draws a line segment connecting point $(2000, 3000)$ to point $(5000, 8000)$. Again he colors the cells whose interiors intersect the segment. How many cells will he color this time?



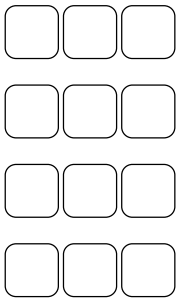
(A) 6000 **(B)** 6500 **(C)** 7000 **(D)** 7500 **(E)** 8000

24. Jean made a piece of stained glass art in the shape of two mountains, as shown in the figure below. One mountain peak is 8 feet high and the other peak is 12 feet high. Each peak forms a 90° angle, and the straight sides of the mountains form 45° angles with the ground. The artwork has an area of 183 square feet. The sides of the mountains meet at an intersection point near the center of the artwork, h feet above the ground. What is the value of h?



(A) 4 **(B)** 5 **(C)** $4\sqrt{2}$ **(D)** 6 **(E)** $5\sqrt{2}$

25. A small airplane has 4 rows of seats with 3 seats in each row. Eight passengers have boarded the plane and are distributed randomly among the seats. A married couple is next to board. What is the probability there will be 2 adjacent seats in the same row for the couple?



(A) $\frac{8}{15}$ **(B)** $\frac{32}{55}$ **(C)** $\frac{20}{33}$ **(D)** $\frac{34}{55}$ **(E)** $\frac{8}{11}$