



MATHEMATICAL ASSOCIATION OF AMERICA

CELEBRATING A CENTURY OF ADVANCING MATHEMATICS

# MAA100

MAA American Mathematics Competitions

31<sup>st</sup> Annual

# AMC 8

American Mathematics Contest 8

Tuesday, November 17, 2015

## INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. For each question, only one answer choice is correct.
3. Mark your answer to each problem on the AMC 8 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. There is no penalty for guessing. Your score is the number of correct answers.
5. Only scratch paper, graph paper, rulers, protractors, and erasers are allowed as aids. Calculators are NOT allowed. No problems on the test *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record your information on the answer form.
8. You will have 40 minutes to complete the test once your proctor tells you to begin.
9. When you finish the exam, *sign your name* in the space provided on the answer form.

The Committee on the American Mathematics Competitions reserves the right to re-examine students before deciding whether to grant official status to their scores. The Committee also reserves the right to disqualify all scores from a school if it determines that the required security procedures were not followed.

The publication, reproduction or communication of the problems or solutions of the AMC 8 during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via copier, telephone, email, internet or media of any type during this period is a violation of the competition rules. After the contest period, permission to make copies of individual problems in **paper form only** for educational use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear the copyright notice.

2015  
**AMC 8**



**DO NOT OPEN UNTIL TUESDAY, NOVEMBER 17, 2015**

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**\*\*ADMINISTRATION ON AN EARLIER DATE  
WILL DISQUALIFY YOUR SCHOOL'S RESULTS\*\***

1. PLEASE READ THE TEACHERS' MANUAL BEFORE NOVEMBER 17, 2015. All rules and instructions needed to administer this exam are contained in the manual. You will not need anything from inside this package until November 17.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 8 CERTIFICATION FORM that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be sent by trackable mail to the AMC office no later than 24 hours following the exam.
4. THE AMC 8 IS TO BE ADMINISTERED DURING A CONVENIENT 40 MINUTE PERIOD. THE EXAM MAY BE GIVEN DURING A REGULAR MATH CLASS.
5. The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via copier, telephone, email, internet or media of any type during this period is a violation of the competition rules.

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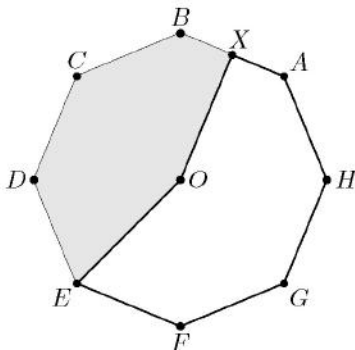
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1. How many square yards of carpet are required to cover a rectangular floor that is 12 feet long and 9 feet wide? (There are 3 feet in a yard.)

(A) 12    (B) 36    (C) 108    (D) 324    (E) 972

2. Point  $O$  is the center of the regular octagon  $ABCDEFGH$ , and  $X$  is the midpoint of side  $\overline{AB}$ . What fraction of the area of the octagon is shaded?

(A)  $\frac{11}{32}$     (B)  $\frac{3}{8}$     (C)  $\frac{13}{32}$     (D)  $\frac{7}{16}$     (E)  $\frac{15}{32}$



3. Jack and Jill are going swimming at a pool that is one mile from their house. They leave home simultaneously. Jill rides her bicycle to the pool at a constant speed of 10 miles per hour. Jack walks to the pool at a constant speed of 4 miles per hour. How many minutes before Jack does Jill arrive?

(A) 5    (B) 6    (C) 8    (D) 9    (E) 10

4. The Centerville Middle School chess team consists of two boys and three girls. A photographer wants to take a picture of the team to appear in the local newspaper. She decides to have them sit in a row with a boy at each end and the three girls in the middle. How many such arrangements are possible?

(A) 2    (B) 4    (C) 5    (D) 6    (E) 12



5. Billy's basketball team scored the following points over the course of the first 11 games of the season:

42, 47, 53, 53, 58, 58, 58, 61, 64, 65, 73.

If his team scores 40 in the 12th game, which of the following statistics will show an increase?

- (A) range    (B) median    (C) mean    (D) mode    (E) mid-range



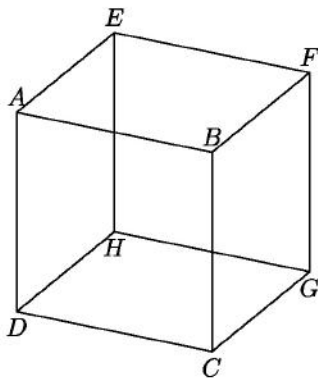
6. In  $\triangle ABC$ ,  $AB = BC = 29$ , and  $AC = 42$ . What is the area of  $\triangle ABC$ ?
- (A) 100    (B) 420    (C) 500    (D) 609    (E) 701
7. Each of two boxes contains three chips numbered 1, 2, 3. A chip is drawn randomly from each box and the numbers on the two chips are multiplied. What is the probability that their product is even?
- (A)  $\frac{1}{9}$     (B)  $\frac{2}{9}$     (C)  $\frac{4}{9}$     (D)  $\frac{1}{2}$     (E)  $\frac{5}{9}$
8. What is the smallest whole number larger than the perimeter of any triangle with a side of length 5 and a side of length 19?
- (A) 24    (B) 29    (C) 43    (D) 48    (E) 57
9. On her first day of work, Janabel sold one widget. On day two, she sold three widgets. On day three, she sold five widgets, and on each succeeding day, she sold two more widgets than she had sold on the previous day. How many widgets in total had Janabel sold after working 20 days?
- (A) 39    (B) 40    (C) 210    (D) 400    (E) 401
10. How many integers between 1000 and 9999 have four distinct digits?
- (A) 3024    (B) 4536    (C) 5040    (D) 6480    (E) 6561

11. In the small country of Mathland, all automobile license plates have four symbols. The first must be a vowel (A, E, I, O, or U), the second and third must be two different letters among the 21 non-vowels, and the fourth must be a digit (0 through 9). If the symbols are chosen at random subject to these conditions, what is the probability that the plate will read "AMC8"?

(A)  $\frac{1}{22,050}$     (B)  $\frac{1}{21,000}$     (C)  $\frac{1}{10,500}$     (D)  $\frac{1}{2,100}$     (E)  $\frac{1}{1,050}$

12. How many pairs of parallel edges, such as  $\overline{AB}$  and  $\overline{GH}$  or  $\overline{EH}$  and  $\overline{FG}$ , does a cube have?

(A) 6    (B) 12    (C) 18    (D) 24    (E) 36



13. How many subsets of two elements can be removed from the set

$$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$$

so that the mean (average) of the nine remaining numbers is 6?

(A) 1    (B) 2    (C) 3    (D) 5    (E) 6

14. Which of the following integers cannot be written as the sum of four consecutive odd integers?

(A) 16    (B) 40    (C) 72    (D) 100    (E) 200

15. At Euler Middle School, 198 students voted on two issues in a school referendum with the following results: 149 voted in favor of the first issue and 119 voted in favor of the second issue. If there were exactly 29 students who voted against both issues, how many students voted in favor of both issues?
- (A) 49    (B) 70    (C) 79    (D) 99    (E) 149



16. In a middle-school mentoring program, a number of the sixth graders are paired with a ninth-grade student as a buddy. No ninth grader is assigned more than one sixth-grade buddy. If  $\frac{1}{3}$  of all the ninth graders are paired with  $\frac{2}{5}$  of all the sixth graders, what fraction of the total number of sixth and ninth graders have a buddy?
- (A)  $\frac{2}{15}$     (B)  $\frac{4}{11}$     (C)  $\frac{11}{30}$     (D)  $\frac{3}{8}$     (E)  $\frac{11}{15}$
17. Jeremy's father drives him to school in rush hour traffic in 20 minutes. One day there is no traffic, so his father can drive him 18 miles per hour faster and gets him to school in 12 minutes. How far in miles is it to school?
- (A) 4    (B) 6    (C) 8    (D) 9    (E) 12



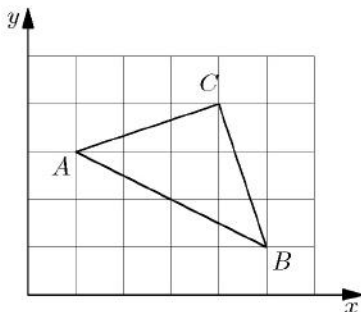
18. An arithmetic sequence is a sequence in which each term after the first is obtained by adding a constant to the previous term. For example, 2, 5, 8, 11, 14 is an arithmetic sequence with five terms, in which the first term is 2 and the constant added is 3. Each row and each column in this  $5 \times 5$  array is an arithmetic sequence with five terms. What is the value of  $X$ ?

(A) 21      (B) 31      (C) 36      (D) 40      (E) 42

1				25
		$X$		
17				81

19. A triangle with vertices at  $A = (1, 3)$ ,  $B = (5, 1)$ , and  $C = (4, 4)$  is plotted on a  $6 \times 5$  grid. What fraction of the grid is covered by the triangle?

(A)  $\frac{1}{6}$       (B)  $\frac{1}{5}$       (C)  $\frac{1}{4}$       (D)  $\frac{1}{3}$       (E)  $\frac{1}{2}$



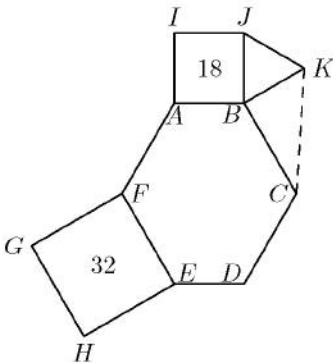
20. Ralph went to the store and bought 12 pairs of socks for a total of \$24. Some of the socks he bought cost \$1 a pair, some of the socks he bought cost \$3 a pair, and some of the socks he bought cost \$4 a pair. If he bought at least one pair of each type, how many pairs of \$1 socks did Ralph buy?

(A) 4    (B) 5    (C) 6    (D) 7    (E) 8



21. In the given figure hexagon  $ABCDEF$  is equiangular,  $ABJI$  and  $FEHG$  are squares with areas 18 and 32 respectively,  $\triangle JBK$  is equilateral and  $FE = BC$ . What is the area of  $\triangle KBC$ ?

(A)  $6\sqrt{2}$     (B) 9    (C) 12    (D)  $9\sqrt{2}$     (E) 32



22. On June 1, a group of students is standing in rows, with 15 students in each row. On June 2, the same group is standing with all of the students in one long row. On June 3, the same group is standing with just one student in each row. On June 4, the same group is standing with 6 students in each row. This process continues through June 12 with a different number of students per row each day. However, on June 13, they cannot find a new way of organizing the students. What is the smallest possible number of students in the group?

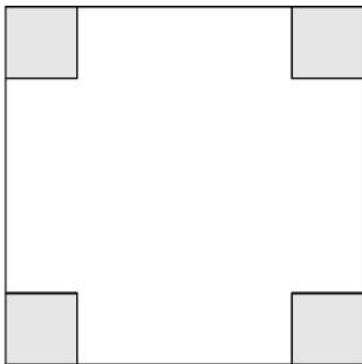
(A) 21    (B) 30    (C) 60    (D) 90    (E) 1080



23. Tom has twelve slips of paper which he wants to put into five cups labeled  $A$ ,  $B$ ,  $C$ ,  $D$ ,  $E$ . He wants the sum of the numbers on the slips in each cup to be an integer. Furthermore, he wants the five integers to be consecutive and increasing from  $A$  to  $E$ . The numbers on the papers are 2, 2, 2, 2.5, 2.5, 3, 3, 3, 3, 3.5, 4, and 4.5. If a slip with 2 goes into cup  $E$  and a slip with 3 goes into cup  $B$ , then the slip with 3.5 must go into what cup?
- (A)  $A$     (B)  $B$     (C)  $C$     (D)  $D$     (E)  $E$
24. A baseball league consists of two four-team divisions. Each team plays every other team in its division  $N$  games. Each team plays every team in the other division  $M$  games with  $N > 2M$  and  $M > 4$ . Each team plays a 76 game schedule. How many games does a team play within its own division?
- (A) 36    (B) 48    (C) 54    (D) 60    (E) 72



25. One-inch squares are cut from the corners of this 5 inch square. What is the area in square inches of the largest square that can be fitted into the remaining space?
- (A) 9    (B)  $12\frac{1}{2}$     (C) 15    (D)  $15\frac{1}{2}$     (E) 17





## SOLUTIONS

Your School Manager will be sent at least one copy of the 2015 AMC 8 Solutions Pamphlet with the report. It is meant to be loaned to students (but not duplicated).

## WRITE TO US

*Comments about the problems and solutions for this AMC 8 should be addressed to:*

**Prof. Norbert Kuenzi, AMC 8 Chair**  
934 Nicolet Ave  
Oshkosh, WI 54901-1634

*Comments about administrative arrangements should be addressed to:*  
MAA American Mathematics Competitions / [amcinfo@maa.org](mailto:amcinfo@maa.org)

MAA American Mathematics Competitions  
PO Box 471  
Annapolis Junction, MD 20701

## AMC 10 & AMC 12

The AMC 10 and AMC 12 are 25-question, 75-minute, multiple choice contests. All schools participating in the AMC 8 receive a brochure and registration form for the 2015 AMC 10. Schools with high scoring students on the AMC 8 should consider administering the AMC 10. The best way to prepare for these contests is to study exams from previous years. Orders for all publications listed below should be addressed to:

MAA American Mathematics Competitions  
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## PUBLICATIONS

A complete listing of the current publications for sale can be found on our web site:  
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