MAA
MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

13th Annual

AMC 10 B

American Mathematics Contest 10 B

Wednesday, February 22, 2012

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 10 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators are allowed. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have **75 minutes** to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

Students who score 120 or above or finish in the top 2.5% on this AMC 10 will be invited to take the 30th annual American Invitational Mathematics Examination (AIME) on Thursday, March 15, 2012 or Wednesday, March 28, 2012. More details about the AIME and other information are on the back page of this test booklet.

2012

AMC 10 B

DO NOT OPEN UNTIL WEDNESDAY, FEBRUARY 22, 2012

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. PLEASE READ THE MANUAL BEFORE FEBRUARY 22, 2011. Nothing is needed from inside this package until February 22.

2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 10 CERTIFICATION FORM (found in the Teachers' Manual) that you followed all rules associated with the conduct of the exam.

3. The Answer Forms must be mailed by trackable mail to the AMC office no later than 24 hours following the exam.

4. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, e-mail, internet or media of any type is a violation of the competition rules.*

The American Mathematics Competitions

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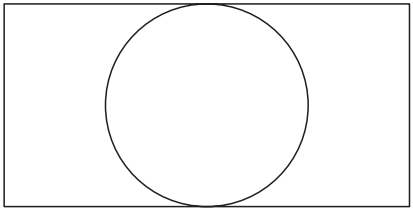
1. Each third-grade classroom at Pearl Creek Elementary has 18 students and 2 pet rabbits. How many more students than rabbits are there in all 4 of the third-grade classrooms?

   (A) 48 **(B)** 56 **(C)** 64 **(D)** 72 **(E)** 80

2. A circle of radius 5 is inscribed in a rectangle as shown. The ratio of the length of the rectangle to its width is $2 : 1$. What is the area of the rectangle?



   (A) 50 **(B)** 100 **(C)** 125 **(D)** 150 **(E)** 200

3. The point in the xy-plane with coordinates $(1000, 2012)$ is reflected across the line $y = 2000$. What are the coordinates of the reflected point?

   (A) $(998, 2012)$ **(B)** $(1000, 1988)$ **(C)** $(1000, 2024)$

   (D) $(1000, 4012)$ **(E)** $(1012, 2012)$

4. When Ringo places his marbles into bags with 6 marbles per bag, he has 4 marbles left over. When Paul does the same with his marbles, he has 3 marbles left over. Ringo and Paul pool their marbles and place them into as many bags as possible, with 6 marbles per bag. How many marbles will be left over?

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

5. Anna enjoys dinner at a restaurant in Washington, D.C., where the sales tax on meals is 10%. She leaves a 15% tip on the price of her meal before the sales tax is added, and the tax is calculated on the pre-tip amount. She spends a total of \$27.50 for dinner. What is the cost of her dinner without tax or tip?

   (A) \$18 **(B)** \$20 **(C)** \$21 **(D)** \$22 **(E)** \$24

6. In order to estimate the value of $x - y$ where x and y are real numbers with $x > y > 0$, Xiaoli rounded x up by a small amount, rounded y down by the same amount, and then subtracted her rounded values. Which of the following statements is necessarily correct?

   (A) Her estimate is larger than $x - y$.

   (B) Her estimate is smaller than $x - y$.

   (C) Her estimate equals $x - y$.

   (D) Her estimate equals $y - x$.

   (E) Her estimate is 0.

7. For a science project, Sammy observed a chipmunk and a squirrel stashing acorns in holes. The chipmunk hid 3 acorns in each of the holes it dug. The squirrel hid 4 acorns in each of the holes it dug. They each hid the same number of acorns, although the squirrel needed 4 fewer holes. How many acorns did the chipmunk hide?

   (A) 30 **(B)** 36 **(C)** 42 **(D)** 48 **(E)** 54

8. What is the sum of all integer solutions to $1 < (x-2)^2 < 25$?

   (A) 10 **(B)** 12 **(C)** 15 **(D)** 19 **(E)** 25

9. Two integers have a sum of 26. When two more integers are added to the first two integers the sum is 41. Finally when two more integers are added to the sum of the previous four integers the sum is 57. What is the minimum number of even integers among the 6 integers?

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

10. How many ordered pairs of positive integers (M, N) satisfy the equation $\frac{M}{6} = \frac{6}{N}$?

    (A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

11. A dessert chef prepares the dessert for every day of a week starting with Sunday. The dessert each day is either cake, pie, ice cream, or pudding. The same dessert may not be served two days in a row. There must be cake on Friday because of a birthday. How many different dessert menus for the week are possible?

    (A) 729 **(B)** 972 **(C)** 1024 **(D)** 2187 **(E)** 2304

12. Point B is due east of point A. Point C is due north of point B. The distance between points A and C is $10\sqrt{2}$ meters, and $\angle BAC = 45°$. Point D is 20 meters due north of point C. The distance AD is between which two integers?

(A) 30 and 31 **(B)** 31 and 32 **(C)** 32 and 33 **(D)** 33 and 34

(E) 34 and 35

13. It takes Clea 60 seconds to walk down an escalator when it is not operating, and only 24 seconds to walk down the escalator when it is operating. How many seconds does it take Clea to ride down the operating escalator when she just stands on it?

(A) 36 **(B)** 40 **(C)** 42 **(D)** 48 **(E)** 52

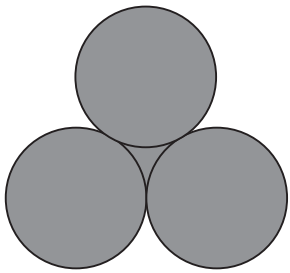
14. Two equilateral triangles are contained in a square whose side length is $2\sqrt{3}$. The bases of these triangles are the opposite sides of the square, and their intersection is a rhombus. What is the area of the rhombus?

(A) $\frac{3}{2}$ **(B)** $\sqrt{3}$ **(C)** $2\sqrt{2}-1$ **(D)** $8\sqrt{3}-12$ **(E)** $\frac{4\sqrt{3}}{3}$

15. In a round-robin tournament with 6 teams, each team plays one game against each other team, and each game results in one team winning and one team losing. At the end of the tournament, the teams are ranked by the number of games won. What is the maximum number of teams that could be tied for the most wins at the end of the tournament?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

16. Three circles with radius 2 are mutually tangent. What is the total area of the circles and the region bounded by them, as shown in the figure?



(A) $10\pi + 4\sqrt{3}$ **(B)** $13\pi - \sqrt{3}$ **(C)** $12\pi + \sqrt{3}$ **(D)** $10\pi + 9$

(E) 13π

17. Jesse cuts a circular paper disk of radius 12 along two radii to form two sectors, the smaller having a central angle of 120 degrees. He makes two circular cones, using each sector to form the lateral surface of a cone. What is the ratio of the volume of the smaller cone to that of the larger?

(A) $\frac{1}{8}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{\sqrt{10}}{10}$ **(D)** $\frac{\sqrt{5}}{6}$ **(E)** $\frac{\sqrt{10}}{5}$

18. Suppose that one of every 500 people in a certain population has a particular disease, which displays no symptoms. A blood test is available for screening for this disease. For a person who has this disease, the test always turns out positive. For a person who does not have the disease, however, there is a 2% false positive rate—in other words, for such people, 98% of the time the test will turn out negative, but 2% of the time the test will turn out positive and will incorrectly indicate that the person has the disease. Let p be the probability that a person who is chosen at random from this population and gets a positive test result actually has the disease. Which of the following is closest to p?

(A) $\frac{1}{98}$ **(B)** $\frac{1}{9}$ **(C)** $\frac{1}{11}$ **(D)** $\frac{49}{99}$ **(E)** $\frac{98}{99}$

19. In rectangle $ABCD$, $AB = 6$, $AD = 30$, and G is the midpoint of $\overline{AD}$. Segment AB is extended 2 units beyond B to point E, and F is the intersection of $\overline{ED}$ and $\overline{BC}$. What is the area of $BFDG$?

(A) $\frac{133}{2}$ **(B)** 67 **(C)** $\frac{135}{2}$ **(D)** 68 **(E)** $\frac{137}{2}$

20. Bernardo and Silvia play the following game. An integer between 0 and 999, inclusive, is selected and given to Bernardo. Whenever Bernardo receives a number, he doubles it and passes the result to Silvia. Whenever Silvia receives a number, she adds 50 to it and passes the result to Bernardo. The winner is the last person who produces a number less than 1000. Let N be the smallest initial number that results in a win for Bernardo. What is the sum of the digits of N?

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

21. Four distinct points are arranged in a plane so that the segments connecting them have lengths a, a, a, a, $2a$, and b. What is the ratio of b to a?

(A) $\sqrt{3}$ **(B)** 2 **(C)** $\sqrt{5}$ **(D)** 3 **(E)** π

22. Let $(a_1, a_2, \ldots, a_{10})$ be a list of the first 10 positive integers such that for each $2 \leq i \leq 10$ either $a_i + 1$ or $a_i - 1$ or both appear somewhere before a_i in the list. How many such lists are there?

(A) 120 **(B)** 512 **(C)** 1024 **(D)** 181,440 **(E)** 362,880

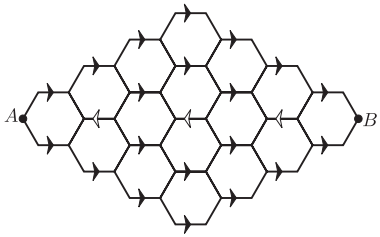
23. A solid tetrahedron is sliced off a solid wooden unit cube by a plane passing through two nonadjacent vertices on one face and one vertex on the opposite face not adjacent to either of the first two vertices. The tetrahedron is discarded and the remaining portion of the cube is placed on a table with the cut surface face down. What is the height of this object?

(A) $\frac{\sqrt{3}}{3}$ **(B)** $\frac{2\sqrt{2}}{3}$ **(C)** 1 **(D)** $\frac{2\sqrt{3}}{3}$ **(E)** $\sqrt{2}$

24. Amy, Beth, and Jo listen to four different songs and discuss which ones they like. No song is liked by all three. Furthermore, for each of the three pairs of the girls, there is at least one song liked by those two girls but disliked by the third. In how many different ways is this possible?

(A) 108 **(B)** 132 **(C)** 671 **(D)** 846 **(E)** 1105

25. A bug travels from A to B along the segments in the hexagonal lattice pictured below. The segments marked with an arrow can be traveled only in the direction of the arrow, and the bug never travels the same segment more than once. How many different paths are there?



(A) 2112 **(B)** 2304 **(C)** 2368 **(D)** 2384 **(E)** 2400

American Mathematics Competitions

WRITE TO US!

Correspondence about the problems and solutions for this AMC 10 and orders for publications should be addressed to:

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The problems and solutions for this AMC 10 were prepared by the MAA's Committee on the AMC 10 and AMC 12 under the direction of AMC 10 Subcommittee Chair:

Dr. Leroy Wenstrom

2012 AIME

The 30th annual AIME will be held on Thursday, March 15, with the alternate on Wednesday, March 28. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score 120 or above or finish in the top 2.5% of the AMC 10, or if you score 100 or above or finish in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the 41st Annual USA Mathematical Olympiad (USAMO) on April 24-25, 2012. The best way to prepare for the AIME and USAMO is to study previous exams. Copies may be ordered as indicated below.

PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site: amc.maa.org